Order of Operations

1

Solve in this order

Brackets	()
Indices	^{2}
Division	÷
Multiplication	×
Addition	+
Subtraction	-

Example

$3 + (5 - 2)^2 \times 4$

$= 3 + (3)^2 \times 4$

$= 3 + 9 \times 4$

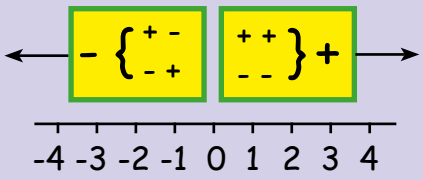
$= 3 + 36$

$= 39$

Positive and Negative Numbers

2

Addition and Subtraction



Examples

-2 + 3 = 1

-2 - 3 = -5

2 - - 3 = 5

2 + - 3 = -1

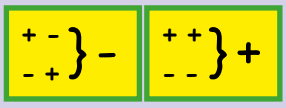
Multiplication and Division

+3 x +2 = +6

-3 x -2 = +6

-3 x +2 = -6

+3 x -2 = -6



+8 ÷ +4 = +2

-8 ÷ -4 = +2

-8 ÷ +4 = -2

+8 ÷ -4 = -2

Prime Numbers

3

A prime number has exactly two factors: 1 and the number itself.

Example
The only factors of 5 are 1 and 5, so 5 is a prime number.

Note:
1 is not a prime number.
2 is the only even prime number.

2, 3, 5, 7, 11, 13, 17, 19...

Square Numbers

4

1, 4, 9, 16, 25, 36 ... n^2

Cube Numbers

1, 8, 27, 64, 125, 216 ... n^3

Triangular Numbers

1, 3, 6, 10, 15, 21 ... $\frac{1}{2}n(n + 1)$

The Fibonacci Sequence

1, 1, 2, 3, 5, 8, 13, ...

Lowest Common Multiple (LCM)

5

The LCM of two or more numbers is the lowest number which is a multiple of all of them.

Example

Multiples of 3: 3, 6, 9, (12) 15, 18, 21, (24)..

Multiples of 4: 4, 8, (12) 16, 20, (24)..

Common multiples of 3 and 4: 12, 24, 36..

The LCM of 3 and 4 is 12.

Highest Common Factor (HCF)

6

The HCF of two or more numbers is the highest number which is a factor of all of them.

Example

Factors of 18: (1), (2), (3), (6), 9 and 18

Factors of 24: (1), (2), (3), 4, (6), 8, 12 and 24

Common factors of 18 and 24: 1, 2, 3 and 6

The HCF of 18 and 24 is 6.

Decimal Places

7

The number of decimal places (d.p.) tells you how many digits are needed after the decimal point.

Examples:

5.4837 is 5.484 to 3 d.p.

5.4837 is 5.48 to 2 d.p.

5.4837 is 5.5 to 1 d.p.

3.697 is 3.70 to 2 d.p.
and 3.7 to 1 d.p.

Significant Figures

8

The number of significant figures (s.f.) shows how many digits are in the number.

eg 7 891 is 8 000 to 1 s.f.
3 032 is 3 030 to 3 s.f.
5.7125 is 5.71 to 3 s.f.
0.002812 is 0.003 to 1 s.f.

Zeros at the beginning and end of a number are not counted as they are only there to keep digits in the correct place value column.

Example

799 is 800 to 1s.f and to 2 s.f.

Standard Form

9

$1 \leq a < 10$

n is a positive or negative whole number.

$a \times 10^{n}$

Examples

$2\,510\,000 = 2.51 \times 10^{6}$

$0.00003105 = 3.105 \times 10^{-5}$

Index Notation

10

Notation	Example
$n^2 = n \times n$	$5^2 = 5 \times 5 = 25$
$n^3 = n \times n \times n$	$2^3 = 2 \times 2 \times 2 = 8$
$n^1 = n$	$9^1 = 9$
$n^0 = 1$	$342^0 = 1$
$(n^a)^b = n^{a \times b} = n^{ab}$	$(7^3)^2 = 7^3 \times 7^3 = 7^6$ $(7^3)^2 = 7^{3 \times 2} = 7^6$

Laws of Indices

11

$$n^a \times n^b = n^{a+b}$$

Example: $7^3 \times 7^2 = 7^{3+2} = 7^5$

$7^3 \times 7^2 = 7 \times 7 \times 7 \times 7 \times 7 = 7^5$

$$\frac{n^a}{n^b} = n^a \div n^b = n^{a-b}$$

Example: $\frac{7^3}{7^2} = 7^3 \div 7^2 = 7^{3-2} = 7^1 = 7$

$7^3 \div 7^2 = \frac{\cancel{7}^{1} \times \cancel{7}^{1} \times 7}{\cancel{7}_{1} \times \cancel{7}_{1}} = 7^1 = 7$

The Unique Factorisation Theorem

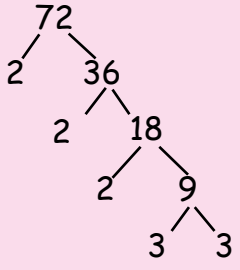
12

Every integer greater than 1 is either a prime number, or is the product of prime numbers.

Example:

$72 = 2 \times 36$
$72 = 2 \times 2 \times 18$
$72 = 2 \times 2 \times 2 \times 9$
$72 = 2 \times 2 \times 2 \times 3 \times 3$

Prime Numbers

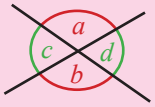


A product is the answer when 2 or more numbers are multiplied together.

Equal Angles

13

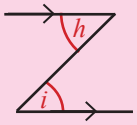
Vertically opposite angles



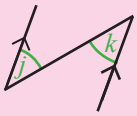
$a° = b°$

$c° = d°$

Alternate angles

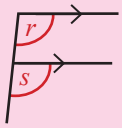


$h° = i°$

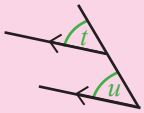


$j° = k°$

Corresponding angles



$r° = s°$

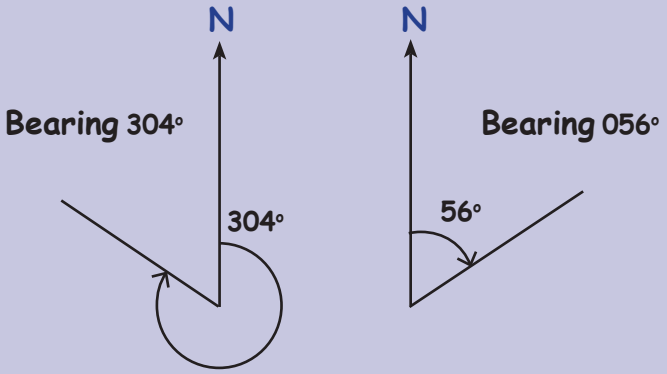


$t° = u°$

Bearings

14

Bearings are measured clockwise from North and have three digits.



The Angle Sum of a Triangle

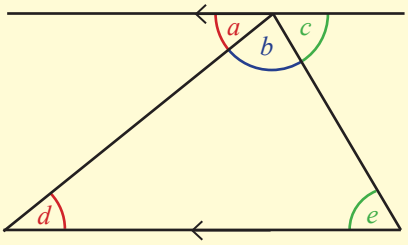
15

Alternate angles are equal so

$a° = d°$

and

$c° = e°$



Angles on a straight line add up to 180°

So $a° + b° + c° = 180°$

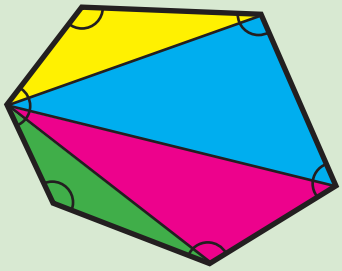
So the interior angles of any triangle add up to 180°

Interior Angles of Polygons

16

A polygon with n sides can be split into $n - 2$ triangles.

The sum of the angles of each triangle is 180°.



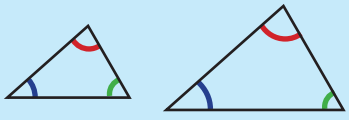
The sum of the interior angles of an n sided polygon is $(n - 2) \times 180°$.

Similar Triangles

17

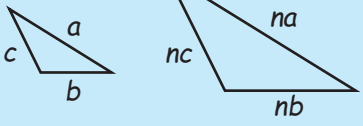
Similar triangles are the same shape but different sizes.

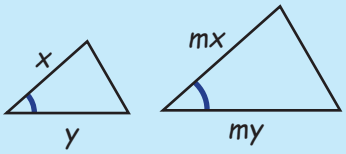
There are 3 ways to spot similar triangles:



corresponding angles all the same (AAA),

corresponding sides in the same proportion (SSS),

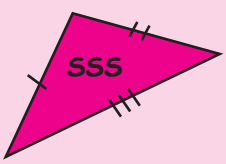


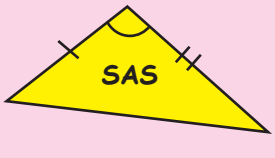


two pairs of sides in same proprtion and included angle equal (SAS).

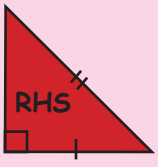
Congruent Triangles

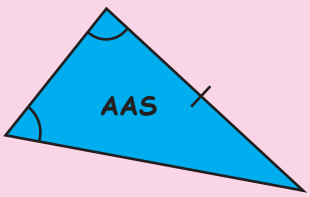
18





Congruent triangles are identical. There are five ways to identify congruent triangles.





Metric Measures

19

10 millimetres (mm) = 1 centimetre (cm)

100 centimetres (cm) = 1 metre (m)

1000 metres (m) = 1 kilometre (km)

1000 grams (g) = 1 kilogram (kg)

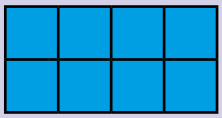
1000 millilitres (ml) = 1 litre (l)

Area

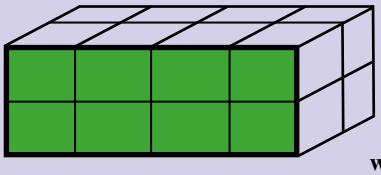
20

Area is measured in square units, units^2 e.g. cm^2, m^2 and km^2.

The **area** of a 2D (flat) shape is the amount of space inside the shape.



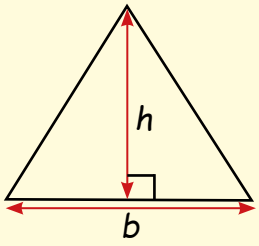
The **surface area** of a 3D (solid) shape is the area of each surface added up.

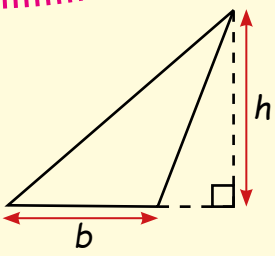


Area of a Triangle 1

21

$$\text{Area } A = \frac{1}{2} b \times h$$

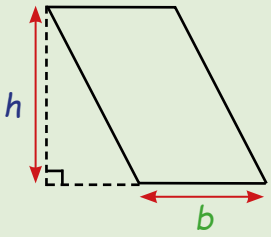


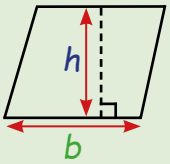


or $\frac{b \times h}{2}$ or $\frac{1}{2}(b \times h)$

Parallelograms and Trapeziums

22



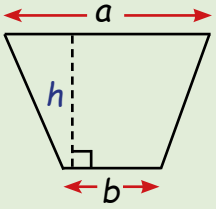


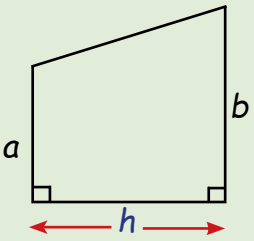
$A = b \times h$

$$A = \frac{a + b}{2} \times h$$

or $\frac{(a + b)h}{2}$

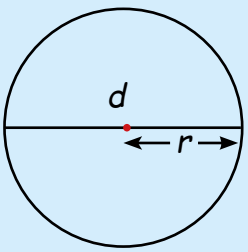
or $\frac{1}{2}(a + b)h$



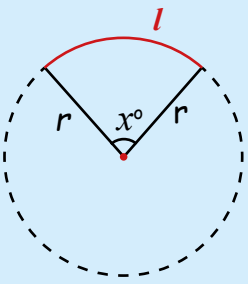


Circumference and Arc Length

23



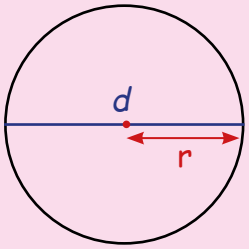
$C = \pi d$ or $C = 2\pi r$

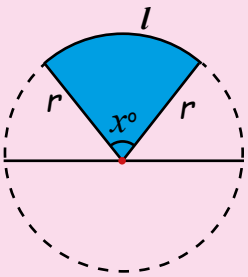


$$l = \frac{x}{360} \times 2\pi r$$

Areas of Circles and Sectors

24



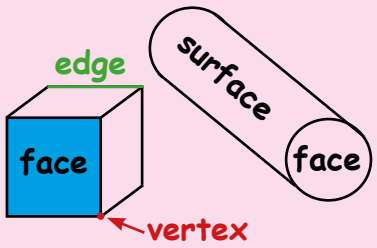


$A = \pi r^2$

$$A = \frac{x}{360} \times \pi r^2$$

Solids

25



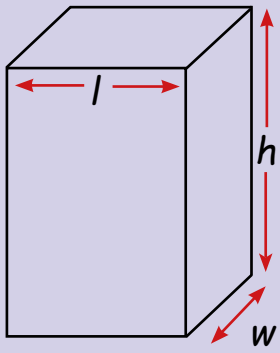
Solid
Cylinder
Cone
Sphere

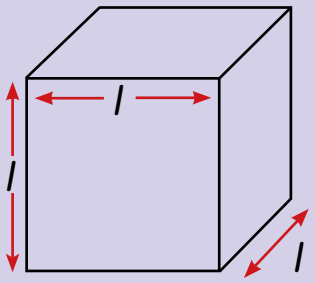
Solid	E	F	V
Cube	12	6	8
Cuboid	12	6	8
Square based pyramid	8	5	5
Triangular prism	9	5	6

Volumes of Cuboids and Cubes

26

Voulume is measured in cubic units, units^3
e.g. cm^3, m^3 and km^3.





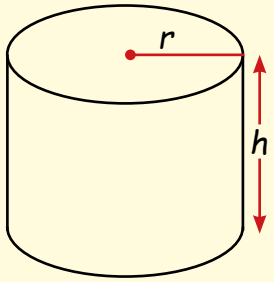
$v = l \times w \times h$

$v = l \times l \times l = l^3$

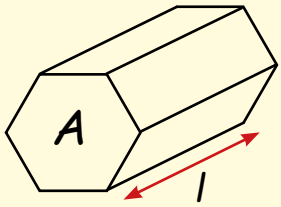
Volumes of Cylinders and Prisms

27

$v = \pi r^2 h$



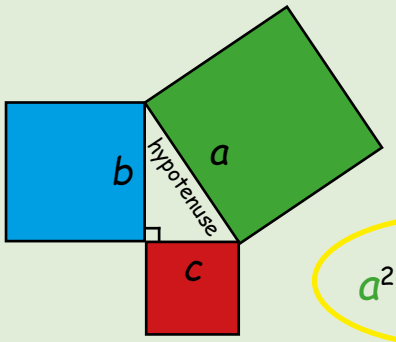
A prism has the same cross sectional area, A, all along its length.



$V = A \times l$

Pythagoras' Theorem

28



$a^2 = b^2 + c^2$

Rearrange to find the shorter sides:

$c^2 = a^2 - b^2$

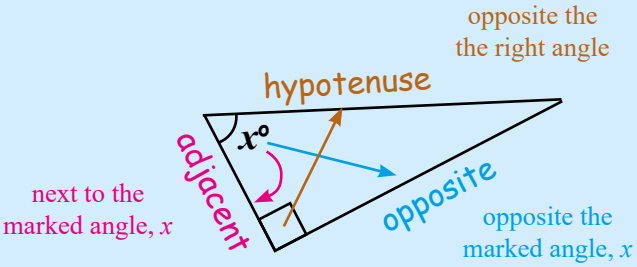
and

$b^2 = a^2 - c^2$

Trigonometry

29

Soh Cah Toa

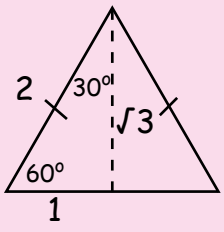


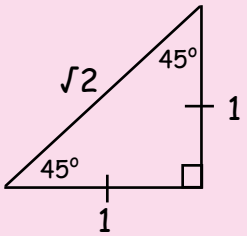
$\text{Sin } x^\circ = \frac{\text{opposite}}{\text{hypotenuse}}$ $\quad \text{Cos } x^\circ = \frac{\text{adjacent}}{\text{hypotenuse}}$

$\text{Tan } x^\circ = \frac{\text{opposite}}{\text{adjacent}}$

Values of Trigonometrical Functions

30

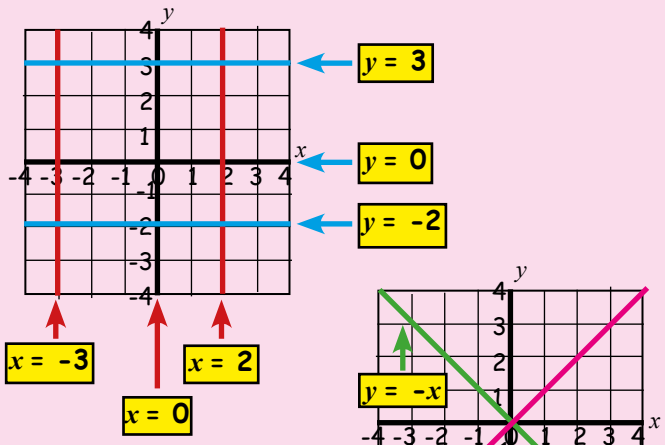




$\sin$	$\cos$	$\tan$
$\sin 0^\circ = 0$	$\cos 0^\circ = 1$	$\tan 0^\circ = 0$
$\sin 30^\circ = \frac{1}{2}$	$\cos 30^\circ = \frac{\sqrt{3}}{2}$	$\tan 30^\circ = \frac{1}{\sqrt{3}}$
$\sin 45^\circ = \frac{1}{\sqrt{2}}$	$\cos 45^\circ = \frac{1}{\sqrt{2}}$	$\tan 45^\circ = 1$
$\sin 60^\circ = \frac{\sqrt{3}}{2}$	$\cos 60^\circ = \frac{1}{2}$	$\tan 60^\circ = \sqrt{3}$
$\sin 90^\circ = 1$	$\cos 90^\circ = 0$	

Straight Lines Graphs

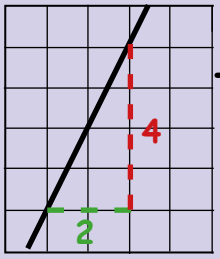
31



The Gradient of a Straight Line

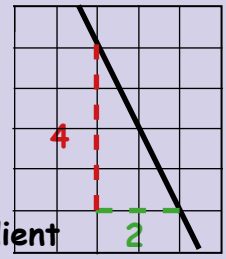
32

Positive Gradient



Gradient $\frac{4}{2} = 2$

Negative Gradient

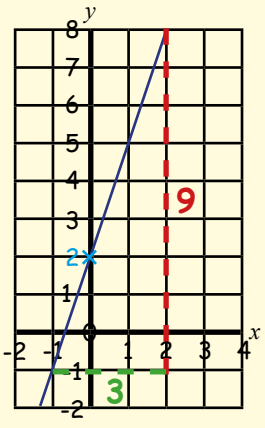


Gradient $-\frac{4}{2} = -2$

$$\text{Gradient, } m = \frac{y_2 - y_1}{x_2 - x_1}$$

Equation of a Straight Line

33



$$y = mx + c$$

Gradient $m = \frac{9}{3} = 3$

y intercept $c = 2$

Equation of the line

$y = mx + c$

$y = 3x + 2$

or use

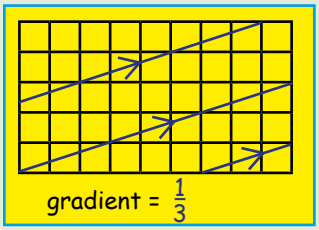
Given the gradient m and a point on the line (x_1, y_1)

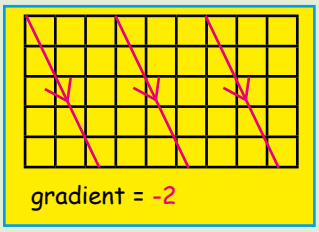
$y - y_1 = m(x - x_1)$

Parallel and Perpendicular Lines

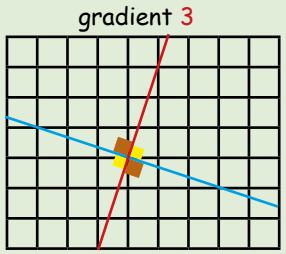
34

Parallel lines have the same gradient.





A line that is perpendicular to a line with a gradient m has gradient $-\frac{1}{m}$



Quadratic Functions

35

Graph of

$y = x^2 + 2x - 3$

or

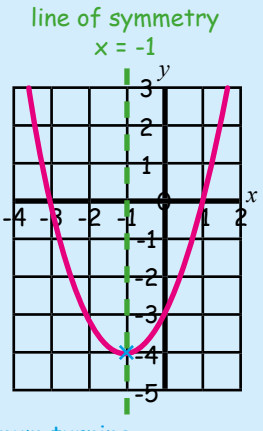
$y = (x - 1)(x + 3)$

Roots of the equation show intersection (crossing) with x-axis

$x^2 + 2x - 3 = 0$

$(x - 1)(x + 3) = 0$

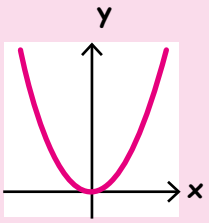
$x = 1$ and $x = -3$



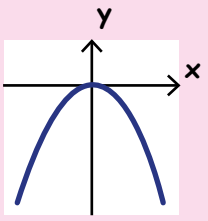
Graph Shapes

36

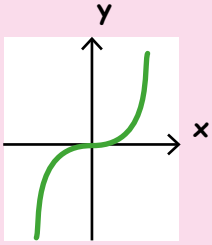
$y = x^2$



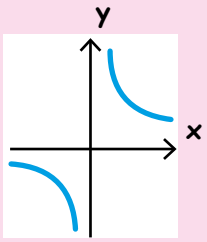
$y = -x^2$



$y = x^3$



$y = \frac{1}{x}$

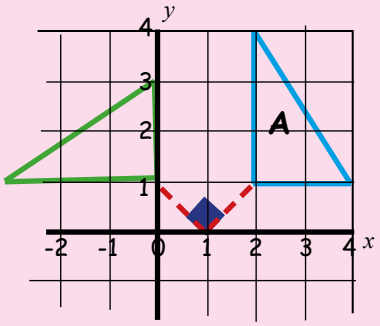


Rotations

37

Rotations are described by
1). **a direction** (clockwise/anticlockwise)
2). **amount of turn** (angle in degrees)
3). **a centre of rotation** (a co-ordinate).

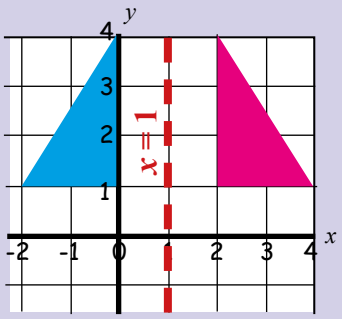
Triangle A is rotated
1). anticlockwise
2). through 90°
3). with centre of rotation (1,0).



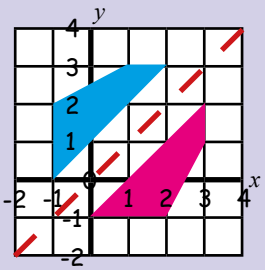
Reflections

38

Reflections are described by a mirror line.



Reflection in the line x = 1.



Reflection in the line y = x.

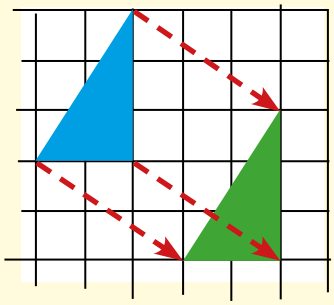
Translations

39

A translation slides an object, without turning it or changing its size.

Vector notation is used to describe translations.

$\begin{pmatrix} x \\ y \end{pmatrix}$ $\begin{pmatrix} - \leftrightarrow + \\ + \updownarrow - \end{pmatrix}$

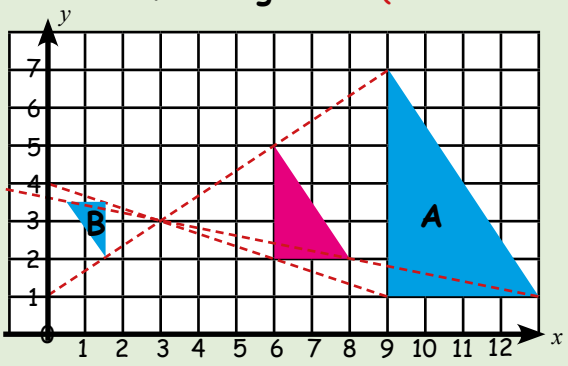


$\begin{pmatrix} 3 \\ -2 \end{pmatrix}$ **means move 3 to the right and 2 down**

Enlargements

40

Enlargements are described by
1). **a scale factor** (a number),
2). **a centre of enlargement** (a co-ordinate).



A is enlargement scale factor 2 about (3, 3)
B is enlargement scale factor $-\frac{1}{2}$ about (3, 3)

Vocabulary for Algebra

41

Word	Examples
Term	$3,\ 9.1,\ 4f,\ g^2,\ -4a^3b^2,\ \sqrt{c}$
Expression	$2a + 5b,\ c^3 - 7,\ p - 4q + 5r$
Equation	$n + 2 = 9,\ m^2 = 25,\ 8d - 2e = 9$
Formula	$A=\pi r^2,\ v = u + at,\ a^2 = b^2 + c^2$
Inequality	$5 > 2,\ \ p + q \geq 7,\ 6 \leq x < 11$
Identity	$4(a + b) \equiv 4a + 4b$ $(u + v)^2 \equiv u^2 + 2uv + v^2$

Sequences (nth Term)

42

Linear

1	2	3	4
9,	11,	13,	15
	+2	+2	+2

2n → 2, 4, 6, 8 (+7 +7 +7 +7)
2n+7 → 5, 8, 11, 14

Quadratic

1	2	3	4
3,	8,	15,	24
	+5	+7	+9
		+2	+2

÷2

1n² → 1, 4, 9, 16 (+2 +4 +6 +8)
n²+2n → 3, 8, 15, 24

The Difference of Two Squares

43

$$a^2 - b^2 = (a - b)(a + b)$$

Examples

$7^2 - 2^2 = (7 - 2)(7 + 2) = 5 \times 9 = 45$

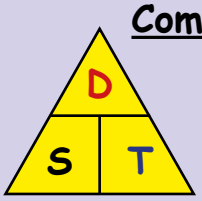
$49^2 - 46^2 = (49 - 46)(49 + 46) = 3 \times 95 = 285$

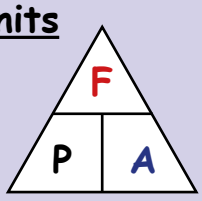
$d^2 - 64 = d^2 - 8^2 = (d - 8)(d + 8)$

$9u^2 - 4v^4 = (3u)^2 - (2v^2)^2 = (3u - 2v^2)(3u + 2v^2)$

Compound Units

44





Speed = $\frac{\text{Distance}}{\text{Time}}$

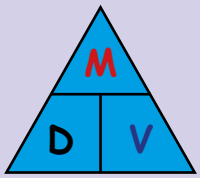
Time = $\frac{\text{Distance}}{\text{Speed}}$

Distance = Speed x Time

Pressure = $\frac{\text{Force}}{\text{Area}}$

Area = $\frac{\text{Force}}{\text{Pressure}}$

Force = Pressure x Area



Density = $\frac{\text{Mass}}{\text{Volume}}$

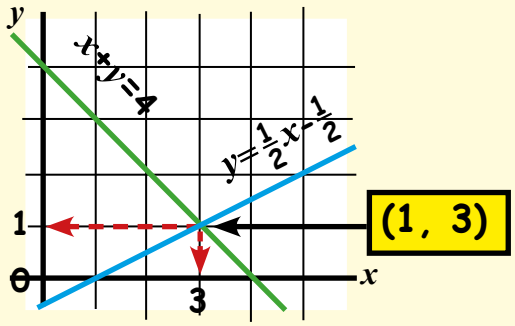
Volume = $\frac{\text{Mass}}{\text{Density}}$

Mass = Density x Volume

Simultaneous Equations 1

45

$x + y = 4$ and $y = \frac{1}{2}x - \frac{1}{2}$



Solution

$x = 1$, $y = 3$

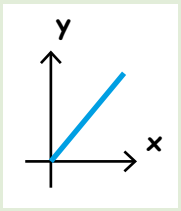
Proportion

46

y is "directly proportional" to x is written

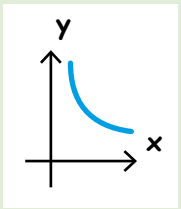
$y \propto x$ or $y = kx$

where k is a constant



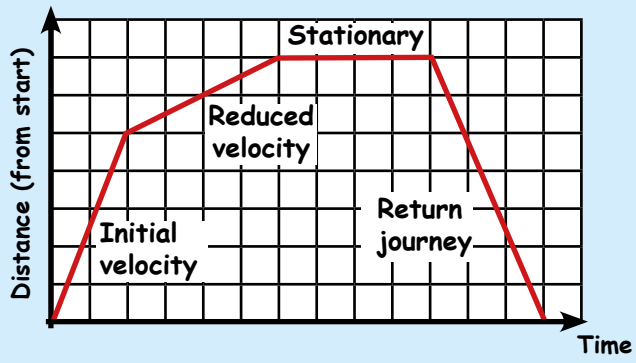
y is "inversely proportional" to x is written

$y \propto \frac{1}{x}$ or $y = \frac{k}{x}$



Distance Time Graphs

47



Gradient = $\frac{y}{x}$ = velocity

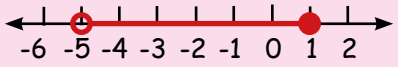
Representing Inequalities

48

Example 1 $-7 < 2x + 3 \leq 5$

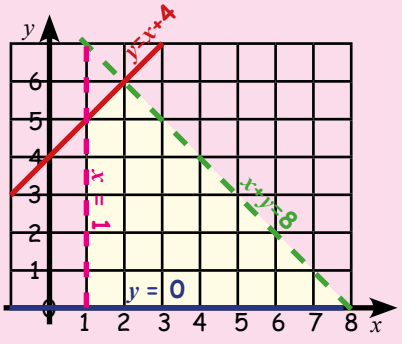
$-10 < 2x \leq 2$ (-3)

$-5 < x \leq 1$ (÷2)



Example 2

$x + y < 8$,
$y \leq x + 4$,
$y \geq 0$ and
$x > 1$.



Types of Data

49

Primary data is data that is collected by you. You do a survey, experiment etc.

Secondary data has been collected by someone else. This may be found in journals, newspapers, the internet etc.

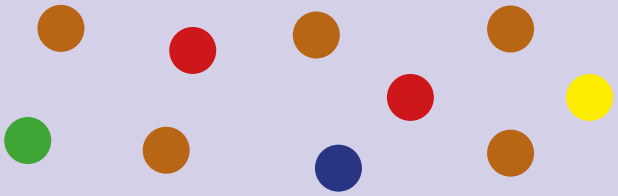
Discrete data can only have certain values. These values jump along the number line e.g. shoe size, number of shoes.

Continuous data can have any value along the number line (within a range) e.g. height, weight, time.

The Mode and Modal Class

50

The mode is the most common.



The modal colour of the spots is brown because there are more brown spots than any other colour.

When information is grouped in classes, the modal class is the most common class.

Median

51

The median is the middle value after the numbers have been *arranged in order*.

Example: 3, 5, 6, 8, 15 — Median 6

If there is no single middle value, the median is the mean of the two middle values.

$\frac{8 + 11}{2} = \frac{19}{2}$ Median 9.5

Example: 6, 7, 8, 11, 14, 17

Finding the Mean

52

To find the mean of a set of values, **add** the values, then **divide** the total by the number of values.

Example — **5 values**

Find the mean of 8, 3, 6, 7 and 6.

$\frac{8 + 3 + 6 + 7 + 6}{5} = \frac{30}{5} = 6$

The mean is **6**.

Note: Beware BIDMAS on a calculator!
Press = after the addition before dividing.

Median from a Frequency Table

53

5th, 6th, 7th and 8th score in here.

Score	5	6	7	8	9	Total
Frequency	1	3	4	6	1	15

1 4 8 14 15

Think of this as a list. The scores are:
5, 6, 6, 6, 7, 7, 7, **7**, 8, 8, 8, 8, 8, 8, 9.

There are 15 scores. The 8th score, **7**, is the median because it is the middle score.

The Mean from a Frequency Table

54

Age, a	Frequency, f	a × f
11	3	11 × 3 = 33
12	5	12 × 5 = 60
13	2	13 × 2 = 26
Total	10	125

Mean age $= \frac{125}{10} = 12.5$

For grouped data (e.g. $120 < h \leq 126$) multiply the **mid-point** (123) of each class by the frequency.

Probability

55

0 The **probability scale** 1

Probability is always written as a **fraction**, a **decimal** or a **percentage**

e.g. $\frac{1}{2}$, 0.5, or 50%.

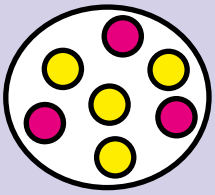
Relative Frequency = $\frac{\text{Number of succesful trials}}{\text{Total number of trials}}$

If the probability of A happening is x, the probability of A **not** happening is $1 - x$.

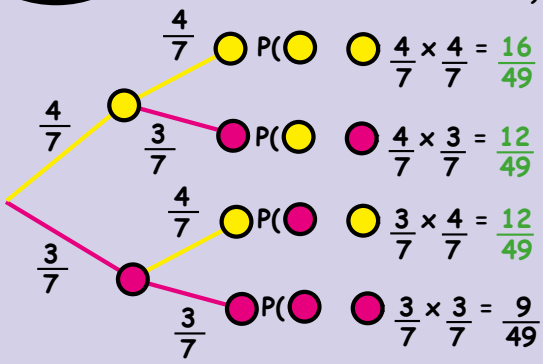
or if $P(A) = x$ then $P(A') = 1 - x$.

Tree Diagrams

56



Independent events
Sampling *with replacement* (a ball is taken, replaced and another ball is chosen).

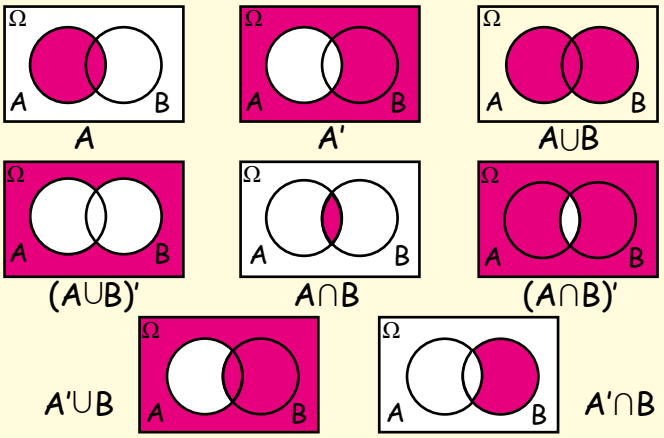


P(At least 1) = $\frac{16}{49} + \frac{12}{49} + \frac{12}{49} = \frac{40}{49}$

Set Notation and Venn Diagrams

57

A' → the complement of A (not A)
$A \cup B$ → the union of A and B (combined)
$A \cap B$ → the intersection of A and B (overlap)
Ω or ξ → the universal set



Dependent and Independent Events

58

When the outcome of one event depends on the outcome of another event, the events are **dependent**.

When the outcome of one event has *no effect* on the outcome of another event, the events are **independent**.

To find the probabilty of two independent events, A and B, happening, **multiply** the probabilities. (AND rule)

P(A and B) = P(A∩B) = P(A) × P(B)

Mutually Exclusive Events

59

If two events cannot happen at the same time they are **mutually exclusive**.

To find the probability of either A or B happening, for mutually exclusive events, **add the probabilities**. (OR rule)

P(A or B) = P(A∪B) = P(A) + P(B)

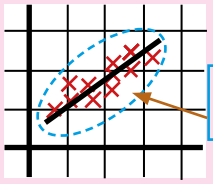
If events are not mutually exclsuive, **add** the probabilities, then **subtract** the probability of both happening.

P(A or B) = P(A∪B) = P(A) + P(B) - P(A and B)

Correlation and Line of Best Fit

60

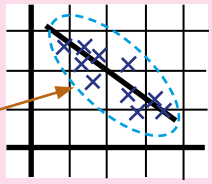
Positive Correlation



As one variable gets bigger, the other one also gets bigger.

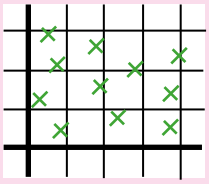
Line of best fit

Negative Correlation



As one variable gets bigger, the other one gets smaller.

No Correlation



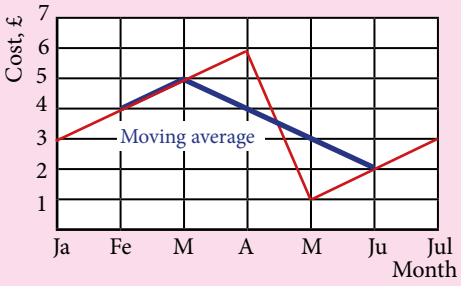
No line of best fit

Moving Average/Time Series

61

3-point moving average - find mean of first 3 pieces of data, move along one - find mean of next 3 pieces of data etc.

Month	Jan	Feb	Mar	Apr	May	Jun	Jul
Cost, £	3	4	5	6	1	2	3
Mov Av		4	5	4	3	2	



Plot at mid-points. Smooths out seasonal variations

Surveys

62

Good Keep it simple
Use tick boxes
Ensure all responses are covered

Bad No leading questions
No personal questions
No overlapping responses

Randon sample: all members of the population have equal chance of being chosen.

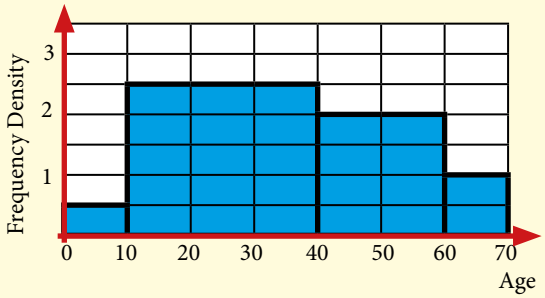
Stratified sample: the population is divided into layers. The same proportion from each layer is surveyed as from the population.

Histograms

63

$$\text{Frequency Density} = \frac{\text{Frequency}}{\text{Class width}}$$

Age	$0 < h \leq 10$	$10 < h \leq 40$	$40 < h \leq 60$	$60 < h \leq 70$
Frequency	5	75	40	10
Frequency Density	5÷10=0.5	75÷30=2.5	40÷20=2	10÷10=1



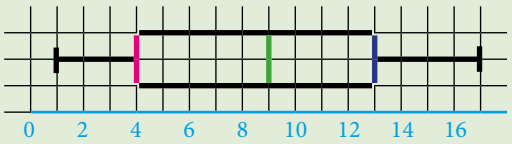
Interquartile Range/Box Plots

64

To find the IQR, order and split into quarters.

1, 1, 4, 5, 7, 9, 11, 13, 13, 15, 17

Lower Quartile, Q_1 Median, Q_2 Upper Quartile, Q_3



IQR = $Q_3 - Q_1$ = 13 – 4 = 9

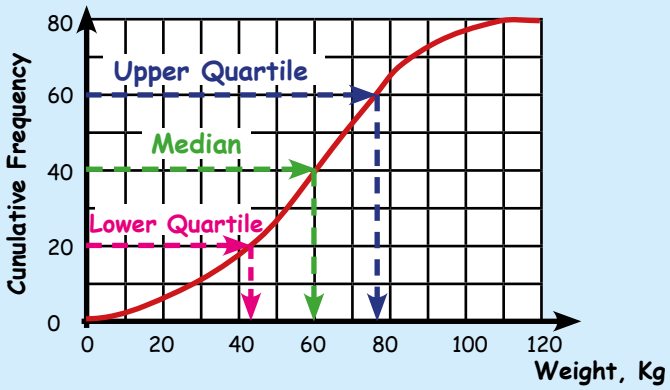
Range = Highest value – lowest = 17 – 1 = 16

IQR discounts extreme or rogue data, by looking at the spread of the *middle* half.

Cumulative Frequency

65

Plot at the top of the class interval.

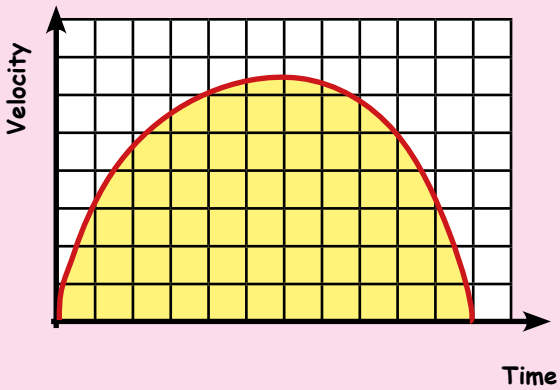


Median = 60 Kg

IQR = $Q_3 - Q_1$ = 76 – 43 = 33 Kg

Velocity/Time Graphs

66



Area = distance travelled

Gradient = acceleration at that point

Recurring Decimals to Fractions

67

Change $0.\dot{1}\dot{9}$ to a fraction.

Let $x = 0.\dot{1}\dot{9}$

$100x = 19.\dot{1}\dot{9}$ (Multiply by 100)

$100x = 19.\dot{1}\dot{9}$

$x = 0.\dot{1}\dot{9}$ (Subtract)

$99x = 19.00$

$x = \frac{19}{99}$ (Rearrange)

Limits of Accuracy

68

Lower bound / Upper bound

Distance

38m to the nearest metre.

$37.5\text{m} \leq d < 38.5\text{m}$

Mass

23kg to the nearest kilogram.

$22.5\text{kg} \leq m < 23.5\text{kg}$

Time

9.8s to the nearest $\frac{1}{10}$ of a second.

$9.75\text{s} \leq m < 9.85\text{s}$

Solving Quadratic Equations

69

Completing the Square

Example: Solve $x^2 + 8x - 5 = 0$

$x^2 + 8x = 5$ (Get x^2 and x terms on own)

$x^2 + 8x + 16 = 5 + 16$ (Divide coefficient of x by 2 and square, add to both sides)

$(x + 4)^2 = 21$

$x + 4 = \pm\sqrt{21}$ Square root both sides

$x = -4 \pm \sqrt{21}$ Get x on its own

$x = 0.58$ or -8.58 (2 d.p.)

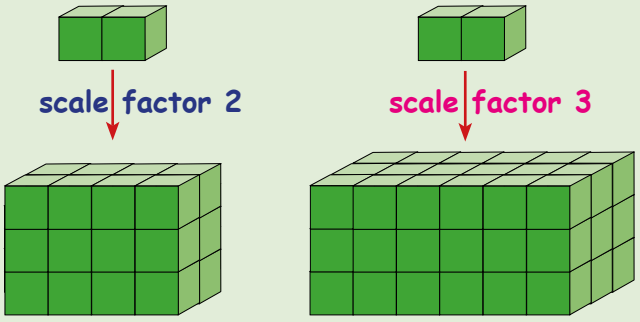
Completing the square leads to the formula.

Formula

$ax^2 + bx + c = 0 \rightarrow \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Scale Factors

70

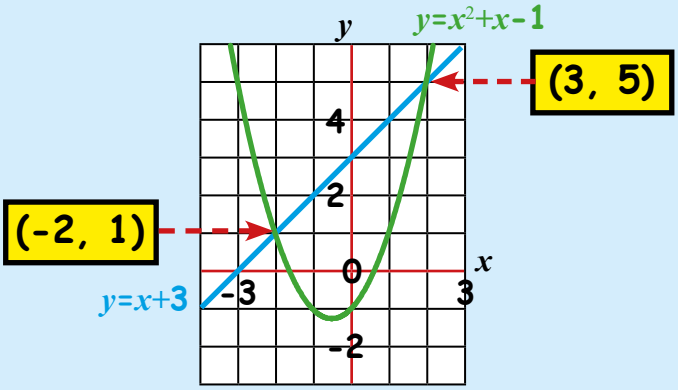


Scale Factor	Length	Area	Volume
2	× 2	× 2^2 = × 4	× 2^3 = × 8
3	× 3	× 3^2 = × 9	× 3^3 = × 27
n	× n	× n^2	× n^3
Units	m	m^2	m^3

Simultaneous Equations 2

71

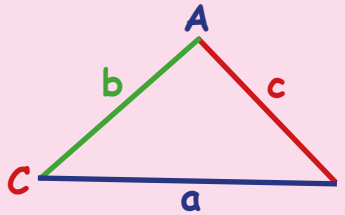
$y = x + 3$ and $y = x^2 + x - 1$



Solutions: (-2, 1) and (3, 5)

The Sine and Cosine Rules

72



Note: sides are named from the angle opposite it.

Cosine Rule

$a^2 = b^2 + c^2 - 2bc\cos A$

$b^2 = a^2 + c^2 - 2ac\cos B$

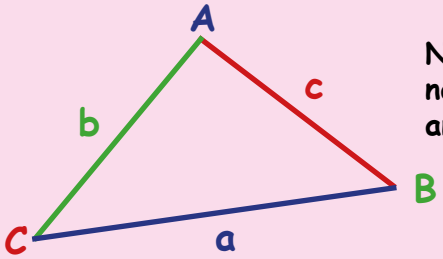
$c^2 = a^2 + b^2 - 2ab\cos C$

Sine Rule

$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ or $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

Area of a Triangle 2

73



Note: sides are named from the angle opposite it.

$$\text{Area} = \frac{1}{2}ab\sin C$$

$$\text{Area} = \frac{1}{2}ac\sin B$$

$$\text{Area} = \frac{1}{2}bc\sin A$$

Surds

74

$$\sqrt{a \times b} = \sqrt{a} \times \sqrt{b}$$

$$\sqrt{90} = \sqrt{9 \times 10} = \sqrt{9} \times \sqrt{10} = 3\sqrt{10}$$

$$\sqrt{a} \times \sqrt{a} = (\sqrt{a})^2 = a$$

$$\sqrt{4} \times \sqrt{4} = (\sqrt{4})^2 = 2 \times 2 = 4$$

$$\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}} \quad \rightarrow \quad \sqrt{\frac{9}{16}} = \frac{\sqrt{9}}{\sqrt{16}} = \frac{3}{4}$$

Index Notation Again

75

$$n^{-1} = \frac{1}{n} \qquad 6^{-1} = \frac{1}{6}$$

$$n^{-x} = \frac{1}{n^x} \qquad 5^{-2} = \frac{1}{5^2} = \frac{1}{25}$$

$$n^{\frac{1}{a}} = \sqrt[a]{n} \qquad 36^{\frac{1}{2}} = \sqrt{36} = 6 \qquad 8^{\frac{1}{3}} = \sqrt[3]{8} = 2$$

$$n^{\frac{a}{b}} = \sqrt[b]{n^a} = (\sqrt{n})^a \quad \text{or} \quad 8^{\frac{2}{3}} = \sqrt[3]{8^2} = \sqrt[3]{64} = 4$$

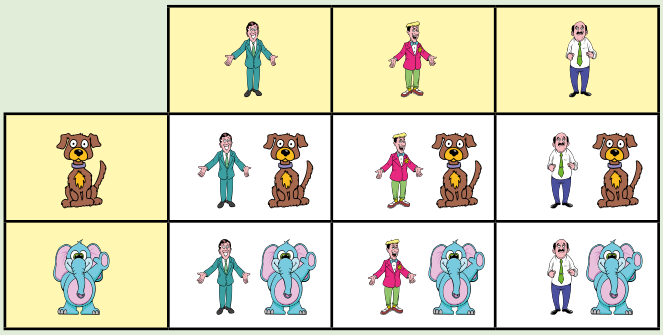
$$8^{\frac{1}{3}} = (\sqrt[3]{8})^2 = 2^2 = 4$$

Product Rule

76

If there are x ways for one event to happen and y ways for another event to happen, then there are $x \times y$ ways for the combination of events to happen.

If there are 3 pet walkers and 2 pets, there are $3 \times 2 = 6$ ways the pets could be walked.



Function Notation

77

$f(x) = 4x + 1$ and $g(x) = x^2$

$f(3) = 4 \times 3 + 1 = 12 + 1 = \underline{13}$ and $g(3) = 3^2 = \underline{9}$

$fg(x) = f(x^2) = 4x^2 + 1$

$gf(x) = g(4x + 1) = (4x + 1)^2 = 16x^2 + 8x + 1$

$fg(3) = f(x^2) = 4x^2 + 1 = 4 \times 3^2 + 1 = 36 + 1 = \underline{37}$

or

$fg(3) = f(3^2) = f(9) = 4 \times 9 + 1 = \underline{37}$

$gf(3) = g(4x + 1) = (4x + 1)^2 = 16x^2 + 8x + 1$

$= 16 \times 3^2 + 8 \times 3 + 1 = \underline{169}$

or

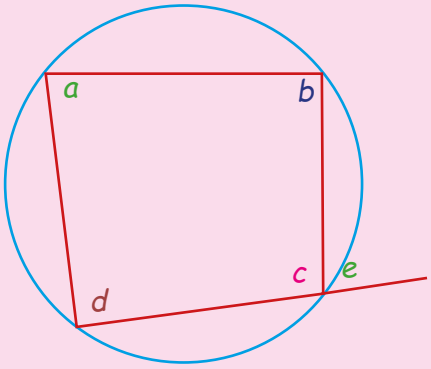
$gf(3) = g(4 \times 3 + 1) = g(13) = 13^2 = \underline{169}$

$f^{-1}(x) = \frac{x - 1}{4}$ and $g^{-1}(x) = \sqrt{x}$

Cyclic Quadrilateral

78

The opposite angles of a cyclic quadrilateral add up to 180°.



$a + c = 180°$.

$b + d = 180°$.

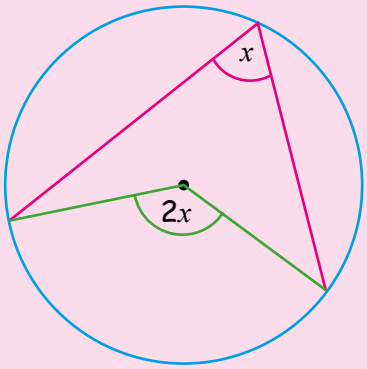
$a = e$.

The exterior angle of a cyclic quadrilateral is equal to the interior opposite angle.

79

Angles at the Centre and Cirumference

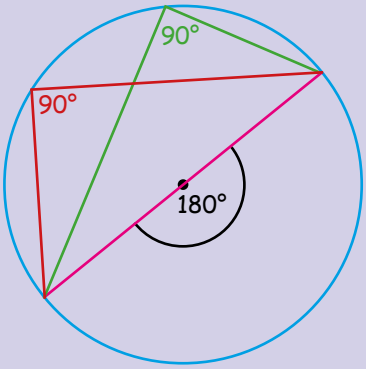
The angle at the centre of a circle is twice that the angle at the circumference.



80

Angles in a semicircle

The angle in a semicircle is a right angle.

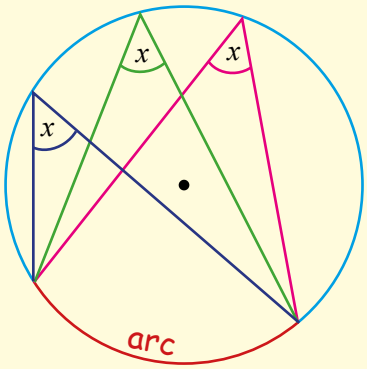


You can also see that the angle at the centre of a circle is twice that the angle at the circumference.

81

Angles in the Same Segment

The angles in the same segment are equal.

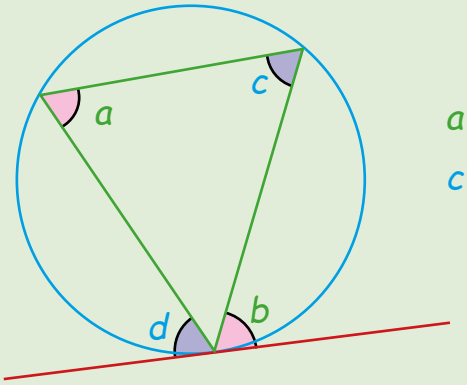


This means that the angles standing on the same arc are equal.

82

The Alternate Segment Theorem

A tangent is a line that just touches a curve or circle at a point, but if extended does not cross it at that point.



$a = b$

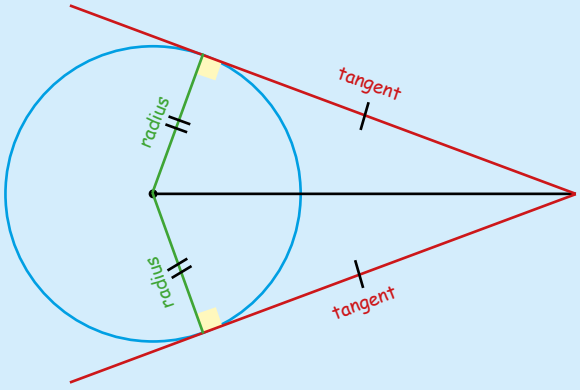
$c = d$

The angle between a tangent and a chord is equal to the angle in the alternate segment.

83

Tangents to a Circle

The two tangents from a point to a circle are equal in length.

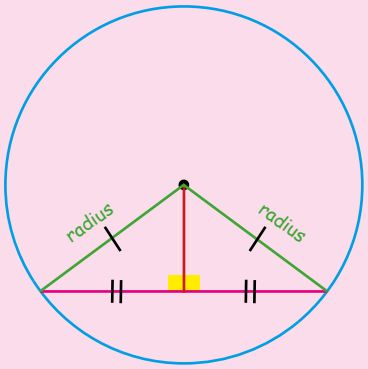


The angle between a tangent and a radius is 90°, a right angle.

84

Chord Bisector

The perpendicular from the centre of a circle to a chord bisects the chord.



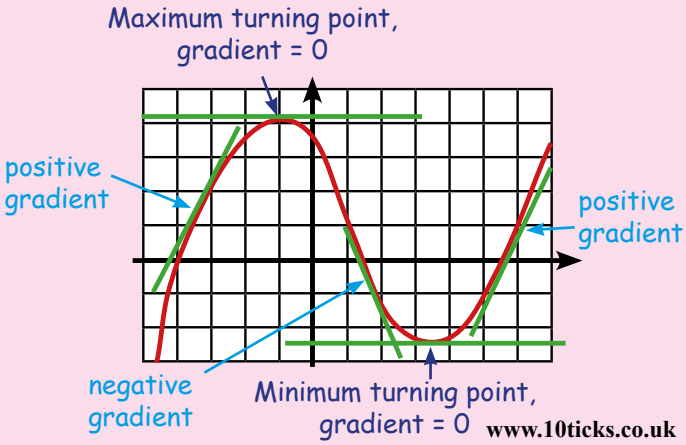
Conversely, if the midpoint of a chord is joined to the centre of a circle, this line is perpendicular to the chord.

Tangents to a Curve

85

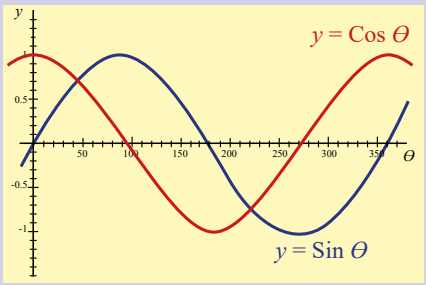
A tangent is a line that touches a curve at one point without crossing the curve.

The gradient of the tangent is the gradient of the curve at that point.



Trigonometrical Graphs

86



The amplitude, a, stretches the graph vertically.

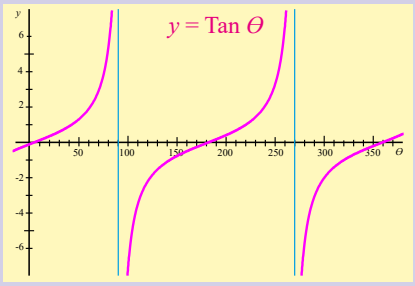
$y = a\text{Sin}\,\theta$, $y = a\text{Cos}\,\theta$.

$y = 2\text{Sin}\,\theta$, would stretch the graph between 2 and -2.

The period, b, stretches the graph horizontally.

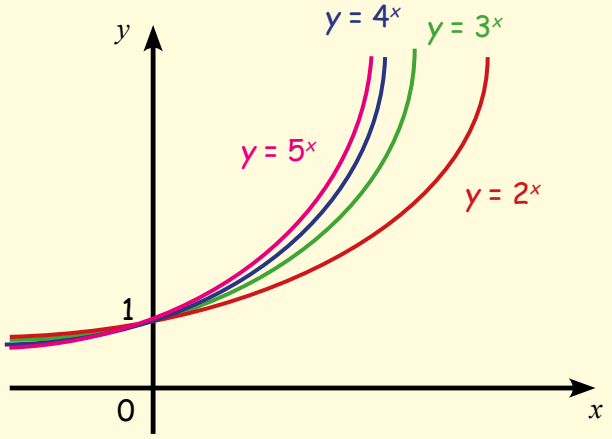
$y = \text{Sin}(b\theta)$, $y = \text{Cos}(b\theta)$.

$y = \text{Sin}\frac{1}{2}\theta$ would stretch the graph between 0 and 720°.



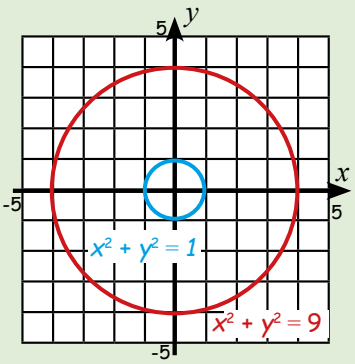
Graphs of $y = K^x$

87



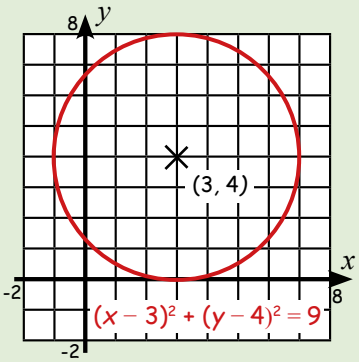
Equation of a Circle

88



If centre is (0, 0) then equation is

$x^2 + y^2 = r^2$



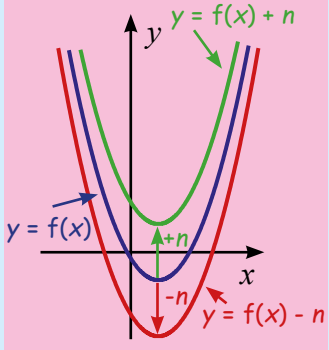
If centre is (a, b) then equation is

$(x - a)^2 + (y - b)^2 = r^2$

Translations of Functions

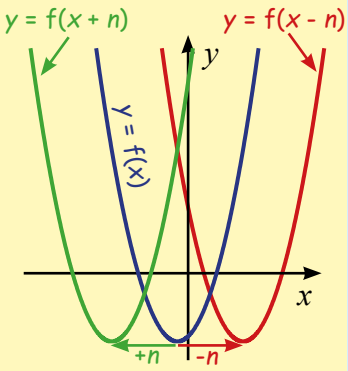
89

$y = f(x) \pm n$



$f(x)$ is moved n units up (+n) or down (-n).

$y = f(x \pm n)$

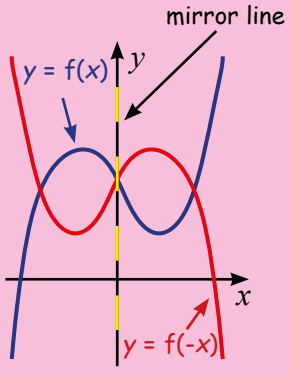


$f(x)$ is moved n units left (+n) or right (-n).

Reflections of Functions

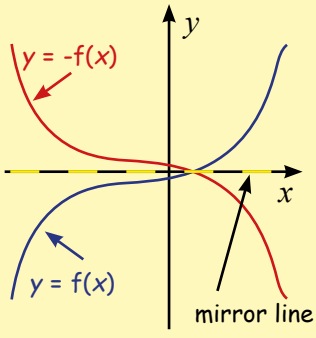
90

$y = f(-x)$



$f(x)$ is reflected in the y-axis.

$y = -f(x)$

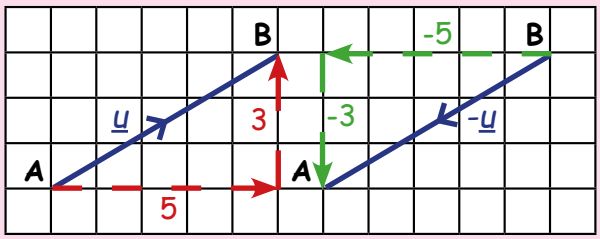


$f(x)$ is reflected in the x-axis.

Vectors Notation

91

A vector has magnitude (length) and direction.



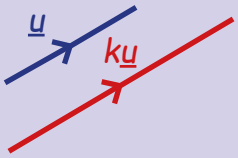
$\overrightarrow{AB} = \underline{u} = \begin{pmatrix} 5 \\ 3 \end{pmatrix}$ $\overrightarrow{BA} = -\underline{u} = \begin{pmatrix} -5 \\ -3 \end{pmatrix}$

Equal vectors are parallel and have **the same** magnitude and direction.

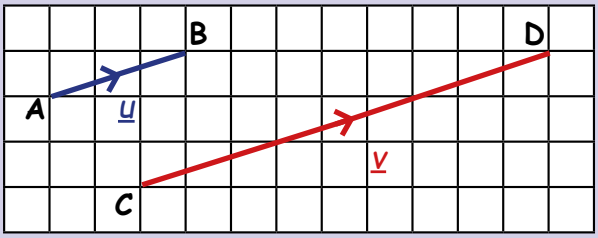
Vectors $\underline{u}$ and $-\underline{u}$ are not equal. They are parallel and have **the same** magnitude, but have opposite directions.

Scalar Multiples (Vectors)

92



Vectors $\underline{u}$ and $k\underline{u}$ are parallel and have the same direction. $k\underline{u}$ is k times the magnitude (length) of $\underline{u}$.

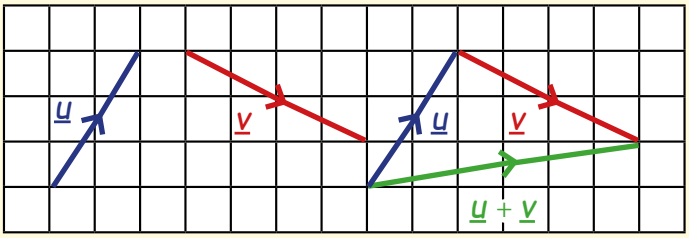


$\overrightarrow{AB} = \underline{u} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$ $\overrightarrow{CD} = \underline{v} = \begin{pmatrix} 9 \\ 3 \end{pmatrix}$ $\underline{u} = 3\underline{v}$

$\underline{v}$ is a scalar multiple of $\underline{u}$.

Adding Vectors

93



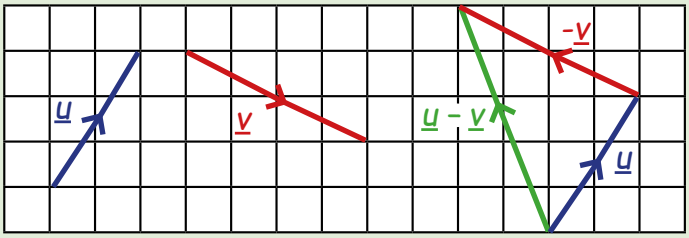
$\underline{u} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$ $\underline{v} = \begin{pmatrix} 4 \\ -2 \end{pmatrix}$

Place the vectors end to end so the arrowheads travel in the same direction. Draw the vector $\underline{u} + \underline{v}$ from the start of $\underline{u}$ to the end of $\underline{v}$.

$$\underline{u} + \underline{v} = \begin{pmatrix} 2 \\ 3 \end{pmatrix} + \begin{pmatrix} 4 \\ -2 \end{pmatrix} = \begin{pmatrix} 2 + 4 \\ 3 + -2 \end{pmatrix} = \begin{pmatrix} 6 \\ 1 \end{pmatrix}$$

Subtracting Vectors

94



$\underline{u} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$ $\underline{v} = \begin{pmatrix} 4 \\ -2 \end{pmatrix}$ so $-\underline{v} = \begin{pmatrix} -4 \\ 2 \end{pmatrix}$

Place $-\underline{v}$ at the end of $\underline{u}$ so the arrowheads travel in the same direction. Draw the vector $\underline{u} - \underline{v}$ from the start of $\underline{u}$ to the end of $-\underline{v}$.

$$\underline{u} - \underline{v} = \begin{pmatrix} 2 \\ 3 \end{pmatrix} - \begin{pmatrix} 4 \\ -2 \end{pmatrix} = \begin{pmatrix} 2 - 4 \\ 3 - -2 \end{pmatrix} = \begin{pmatrix} -2 \\ 5 \end{pmatrix}$$

Tick the boxes when you know the facts.

1 ☐	13 ☐	25 ☐	37 ☐	49 ☐	61 ☐	73 ☐	85 ☐
2 ☐	14 ☐	26 ☐	38 ☐	50 ☐	62 ☐	74 ☐	86 ☐
3 ☐	15 ☐	27 ☐	39 ☐	51 ☐	63 ☐	75 ☐	87 ☐
4 ☐	16 ☐	28 ☐	40 ☐	52 ☐	64 ☐	76 ☐	88 ☐
5 ☐	17 ☐	29 ☐	41 ☐	53 ☐	65 ☐	77 ☐	89 ☐
6 ☐	18 ☐	30 ☐	42 ☐	54 ☐	66 ☐	78 ☐	90 ☐
7 ☐	19 ☐	31 ☐	43 ☐	55 ☐	67 ☐	79 ☐	91 ☐
8 ☐	20 ☐	32 ☐	44 ☐	56 ☐	68 ☐	80 ☐	92 ☐
9 ☐	21 ☐	33 ☐	45 ☐	57 ☐	69 ☐	81 ☐	93 ☐
10 ☐	22 ☐	34 ☐	46 ☐	58 ☐	70 ☐	82 ☐	94 ☐
11 ☐	23 ☐	35 ☐	47 ☐	59 ☐	71 ☐	83 ☐	
12 ☐	24 ☐	36 ☐	48 ☐	60 ☐	72 ☐	84 ☐	

How to use these revision cards

These cards are for Higher GCSE.

Print out the revision cards in colour. Use the thickest paper your printer can take.

On the back of the cards make notes or write typical exam questions for this fact.

When you know the fact, tick the box for that card. Each card is numbered.

The cards are one small part of the revision process and only cover the key facts.

The best practise is to go through as many past papers as possible. For past papers go to www.10ticks.co.uk/revision/GCSE.

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5). Type in the unique school code into the '**Affiliate Code**' box and click Apply
6). Check that the price has been correctly updated and press **Checkout**
7). Ensure your personal details are correct and press **Confirm Details**
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