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GCSE Mathematics

Foundation Tier

Stafford Burndred

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GCSE Mathematics

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Contents

Contents

Introduction

The aim of this guide is to ensure you pass your exam and maybe even achieve a higher grade than you expect to. Ask your teacher to explain any points that you don't understand. You will have to work hard at your revision. Just reading this book will not be enough. You should also try to work through the diagnostic tests at the back and any past papers that your teacher might set you to ensure that you get enough practice.

Remember it is your guide, so you may decide to personalise it, make notes in the margin, use the checklist in the contents to assess your progress, etc.

You may also find it useful to mark or highlight important sections, pages or questions you find difficult. You can then look at these sections again later.

The guide is divided into over 60 short topics to make it easy to revise. Try to set aside time every week to do some revision at home.

The guide is pocket-sized to make it easy to carry. Use it wherever you have time to spare, eg registration, break, etc.

Using the guide

It may help you to place a blank piece of paper over the answers. Then read the notes and try the questions.

Do your working out and answers on the blank piece of paper. Don't just read the answers. Compare your answers with the worked answer. If your answer is wrong read the page again and then mark or make a note of the question or page. You will need to try the question again at a later date.

If you need to look up a topic to revise, try using the contents pages, or even better, the index at the back of the book.

The diagnostic tests

Diagnostic tests and answers are provided at the back of the book. You should use these to identify your weaknesses.

The author has been teaching at this level for over 20 years and is an experienced examiner.

Examiner's tips

Success in exams depends in no small part on how you approach the actual papers on the day. The following suggestions are designed to improve your exam technique.

- Read the instructions on the paper carefully.
- If you only have to answer some of the questions, read the questions and choose which to do.
- If the instructions say "Answer all the questions", work steadily through the paper, leaving out any questions you cannot do. Return to these later.
- Read each question carefully to be sure what it is you are required to do.
- If your examination includes an oral test, be sure to follow the instructions and listen carefully. For some parts you must write down only the answer – no working!
- Set out all your work carefully and neatly and make your method clear. If the examiner can see what you have done, they will be able to give marks for the correct method even if you have the wrong answer.
- If you have to write an explanation as your answer, try to keep it short.
- There will be a list of formulae at the front of the question paper. Make sure you know what is on it, and what is not – you will have to remember those!
- Check your answers, especially numerical ones. Look to see if your answers are sensible.
- Make sure you know how to use your calculator. They don't all work in the same way. Use the instruction book for your calculator when you are learning but don't take it into the exam.
- When doing a calculation, keep all the figures shown on your calculator until the end. Only round off the final answer.
- Sometimes, in a later part of a question, you need to calculate using an earlier answer. Use all the figures in the calculator display. If you use a rounded answer it could cause an error.
- Make sure you take all the equipment you may need to the exam: pens, pencils, rubber, ruler, compasses, angle measurer and calculator – make sure that the battery is working.
- When you have completed the exam, check to see that you have not missed out any questions, especially on the back page.

Exam questions often use these words:

"*Show your working*"
You must show your working. If you give a correct answer without working you will receive no marks.

"*Do not use a calculator*"
You must show enough working to convince the examiner that you have not used a calculator. (But you should still check your answer with a calculator.)

"*Check using an approximation*" or "*Estimate*" or "*Give an approximate answer*"
You must show your method and working.

"*Compare*"
If you are asked to compare two sets of data you must refer to both sets of data and not just one set.

Avoiding panic

If you have done your revision you have no need to panic. If you find the examination difficult, so will everyone else. This means that the pass mark will be lower.

If you cannot do a question, move on and don't worry about it. Often the answer will come to you a few minutes later.

If panic occurs, try to find a question you can do. Success will help to calm your nerves.

The consultant editor is at the very hub of setting and marking GCSE Mathematics, being Chairman of Examiners after many years as a Chief Examiner.

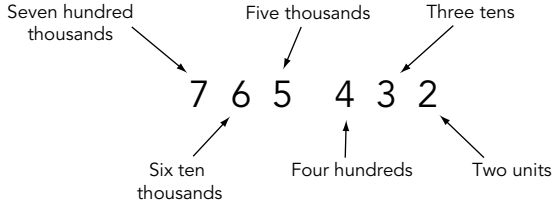
Number skills

We use numbers every day of our lives. You need to be confident in the basic number skills.

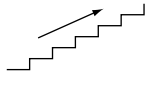
Place value

You need to be able to write numbers in words and write lists of numbers in order of size.

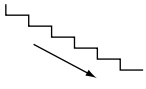
765 432 – This means seven hundred and sixty-five thousand, four hundred and thirty-two.



Ascending order means going up.
(small to large)



Descending order means going down.
(large to small)



1 000 000 This is one million
500 000 This is half of a million
250 000 This is a quarter of a million

Questions

1 Write the following numbers in words:
   a 5020
   b 36 240

2 Place the following numbers in ascending order:
   5372 6843 1998 9081 876

3 Place the following numbers in descending order:
   382 4654 2838 4009 2816

Answers

1 a Five thousand and twenty
   b Thirty-six thousand, two hundred and forty

2 876, 1998, 5372, 6843, 9081

3 4654, 4009, 2838, 2816, 382

Multiplication facts

You must learn your multiplication tables and know how to multiply and divide by 10, 100 and 1000. You must learn these multiplication tables:

1 x 2 = 2	1 x 3 = 3	1 x 4 = 4	1 x 5 = 5	1 x 6 = 6
2 x 2 = 4	2 x 3 = 6	2 x 4 = 8	2 x 5 = 10	2 x 6 = 12
3 x 2 = 6	3 x 3 = 9	3 x 4 = 12	3 x 5 = 15	3 x 6 = 18
4 x 2 = 8	4 x 3 = 12	4 x 4 = 16	4 x 5 = 20	4 x 6 = 24
5 x 2 = 10	5 x 3 = 15	5 x 4 = 20	5 x 5 = 25	5 x 6 = 30
6 x 2 = 12	6 x 3 = 18	6 x 4 = 24	6 x 5 = 30	6 x 6 = 36
7 x 2 = 14	7 x 3 = 21	7 x 4 = 28	7 x 5 = 35	7 x 6 = 42
8 x 2 = 16	8 x 3 = 24	8 x 4 = 32	8 x 5 = 40	8 x 6 = 48
9 x 2 = 18	9 x 3 = 27	9 x 4 = 36	9 x 5 = 45	9 x 6 = 54
10 x 2 = 20	10 x 3 = 30	10 x 4 = 40	10 x 5 = 50	10 x 6 = 60

1 x 7 = 7	1 x 8 = 8	1 x 9 = 9	1 x 10 = 10
2 x 7 = 14	2 x 8 = 16	2 x 9 = 18	2 x 10 = 20
3 x 7 = 21	3 x 8 = 24	3 x 9 = 27	3 x 10 = 30
4 x 7 = 28	4 x 8 = 32	4 x 9 = 36	4 x 10 = 40
5 x 7 = 35	5 x 8 = 40	5 x 9 = 45	5 x 10 = 50
6 x 7 = 42	6 x 8 = 48	6 x 9 = 54	6 x 10 = 60
7 x 7 = 49	7 x 8 = 56	7 x 9 = 63	7 x 10 = 70
8 x 7 = 56	8 x 8 = 64	8 x 9 = 72	8 x 10 = 80
9 x 7 = 63	9 x 8 = 72	9 x 9 = 81	9 x 10 = 90
10 x 7 = 70	10 x 8 = 80	10 x 9 = 90	10 x 10 =100

If you multiply a whole number by 10 add one nought. For example, 24 x 10 = 240

If you multiply a whole number by 100 add two noughts. For example, 82 x 100 = 8200

If you divide a whole number by 10 remove one nought. For example, 360 ÷ 10 = 36

If you divide a whole number by 100 remove two noughts. For example, 7200 ÷ 10 = 72

Questions

1 What is the missing number?

a 3 x ☐ = 24 b ☐ ÷ 4 = 7 c 30 ÷ ☐ = 6

2 Work out the following:

a 36 x 10 = b 478 x 100 = c 3200 ÷ 100 =

Answers

1 a 8, b 28, c 5

2 a 360, b 47 800, c 32

Negative numbers

For easy examples think of a thermometer. For difficult numbers learn how to use the [+/–] key on your calculator. For example:

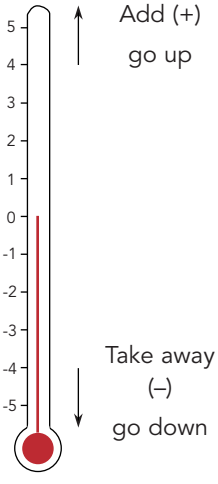
4 is a larger number than 3

3 is a larger number than 2

But look what happens when numbers are negative:

-3 is a larger number than -4

-4 is a larger number than -5



Try these:

-4 + 3 Start at -4 Add 3 means go up 3 = -1

2 – 5 Start at 2 Take away 5 means go down 5 = -3

Questions

1 Place these numbers in order, largest first:

-3 6 -7 0 4

2 -6 + 8 =

3 -1 – 4 =

Answers

1 6, 4, 0, –3, –7

2 Start at –6 go up 8 = 2

3 Start at –1 go down 4 = –5

Using a calculator for negative numbers

First find the [+/–] key.

Question	Calculator sequence	Answer
-3 + 7 =	3 [+/–] + 7 =	4
-5 – -6 =	5 [+/–] – 6 [+/–] =	1
-4 + -3 =	4 [+/–] + 3 [+/–] =	-7
-3 x -4 =	3 [+/–] x 4 [+/–] =	12
4 x -7 =	4 x 7 [+/–] =	-28
$\frac{-6}{-2}$ =	6 [+/–] ÷ 2 [+/–] =	3

Solving problems without a calculator

Sometimes you will be asked to work out answers without a calculator.

If the examination question states "Do not use a calculator" or "Show your working", you must show all of your working. If you do not you will lose marks.

Advice: Check your answer with a calculator. If the calculator answer disagrees with your answer, then check your answer.

Answering problems

Read the question carefully. Try to decide if it is an add, take away, multiply or divide question.

If the numbers are large and you do not know if you should add, take away, multiply or divide try putting easier numbers in the question to help you decide.

Questions

1 Work out the following:

a
$$\begin{array}{r} 36 \\ \times\ 8 \\ \hline \\ \hline \end{array}$$

b $92 \div 4$

c
$$\begin{array}{r} 827 \\ +\ 189 \\ \hline \\ \hline \end{array}$$

d
$$\begin{array}{r} 472 \\ -\ 139 \\ \hline \\ \hline \end{array}$$

2 48 sweets were divided equally between 8 girls.
How many sweets did each girl receive?

Answers

1 a
$$\begin{array}{r} 36 \\ \times\ 8 \\ \hline 288 \\ \hline {\scriptstyle 4} \end{array}$$
You must show the 4. If you do not you will lose marks.

b
$$4\overline{)\,9^{1}2}\ \begin{array}{l} 23 \end{array}$$
You must show the 1.

c
$$\begin{array}{r} 827 \\ +\ 189 \\ \hline 1016 \\ \hline {\scriptstyle 1\ 1\ 1} \end{array}$$
You must show the numbers you carry.

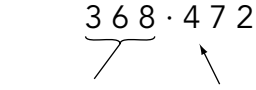
d
$$\begin{array}{r} {\scriptstyle 6\ \ 1} \\ 4\not{7}{}^{1}2 \\ -\ 139 \\ \hline 333 \end{array}$$
You must show your method.

2 $48 \div 8 = 6$

Calculation checks

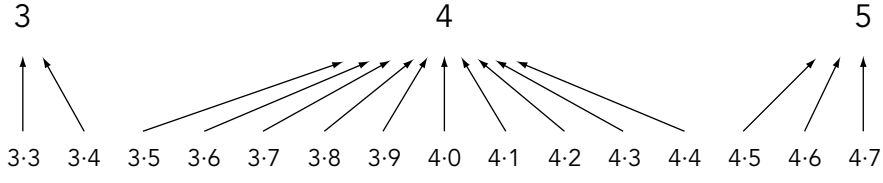
We can work out an approximate answer by estimating numbers. This is useful if you need to check that your answer is sensible.

Whole numbers are numbers to the left of the decimal point.



whole numbers 4 is the first number after the decimal point

When writing a number correct to the nearest whole number you must look at the first number after the decimal point.



If the first number after the decimal point is 5 or more (ie 5, 6, 7, 8, or 9) then the whole number increases by 1.

If the first number after the decimal point is 4 or less (ie 0, 1, 2, 3, or 4) then the whole number stays the same.

376.824 written to the nearest whole number is 377.
428.374 written to the nearest whole number is 428.

Question

Estimate the total amount of money:
£3128 + £6280 +£5972 + £5310 + £8089

Answer

If the question asks for an estimate you must approximate each number. You must show all of your working.

£3128 is approximately	£3000
£6280 is approximately	£6000
£5972 is approximately	£6000
£5310 is approximately	£5000
£8089 is approximately	£8000
Total	£28 000

You must not add the numbers with a calculator and then approximate that answer. Look what happens:

£3128 + £6280 +£5972 + £5310 + £8089 = £28 779

£28 779 is approximately £29 000

If you write the answer £29 000 without working you will obtain no marks.

Maths without a calculator

We had a power cut in my local supermarket. Unbelievably it had to close because no-one knew how to add up and work out change without a cash register! In the following section you will be shown all of the old methods and some shortcuts too.

If a question states "do not use a calculator" or "estimate" you **must** show your working because a correct answer without working will earn **no** marks.

Remember: Always check your answer with a calculator

Questions in this section *Maths without a calculator* should be revised before your aural test, ie mental arithmetic test. In an aural test, where you are not allowed a calculator, working is not required.

Ask your teacher if your GCSE exam includes an aural test

Mental arithmetic shortcuts – 1

Large numbers with several noughts on the end are very clumsy to work with. It is easier to take away the noughts and work with smaller numbers.

Multiplying and dividing whole numbers by 10, 100, 1000

You know 3 x 4 = 12

To work out 30 x 400 multiply 3 x 4 then add the noughts

$$30 \times 400 = 12000$$

1 nought + 2 noughts = 3 noughts

You know 21 ÷ 7 = 3

To work out 21000 ÷ 70 divide 21 ÷ 7 then take away the noughts

$$21000 \div 70 = 300$$

3 noughts – 1 nought = 2 noughts

Questions

1 4800 ÷ 60 =

2 7000 x 40 =

Answers

1 4800 ÷ 60 = 80 Answer 80

2 noughts – 1 nought = 1 nought

2 7000 x 40 = 280 000 Answer 280 000

3 noughts + 1 nought = 4 noughts

Mental arithmetic shortcuts – 2

These shortcuts are very useful in everyday life. Using metric units (eg for length, mass, capacity) is easier if you can multiply and divide by 10, 100 and 1000.

The number of noughts tells you how many places to move the decimal point.
1 nought = 1 place, 2 noughts = 2 places, 3 noughts = 3 places, etc.

Multiplying and dividing decimals by 10, 100, 1000

Multiplying and dividing by 10, 100, 1000 can be worked out without using a calculator.

Multiplying

By 10 Move the decimal point one place to the right

$$3{\cdot}874 \times 10 = 3874 = 38{\cdot}74$$

By 100 Move the decimal point two places to the right

$$64{\cdot}3 \times 100 = 6430 = 6430$$

By 1000 Move the decimal point three places to the right

$$7{\cdot}2694 \times 1000 = 72694 = 7269{\cdot}4$$

Dividing

By 10 Move the decimal point one place to the left

$$58{\cdot}2 \div 10 = 582 = 5{\cdot}82$$

By 100 Move the decimal point two places to the left

$$43{\cdot}62 \div 100 = 04362 = 0{\cdot}4362$$

By 1000 Move the decimal point three places to the left

$$2{\cdot}85 \div 1000 = 00285 = 0{\cdot}00285$$

Questions

1 3.61 x 100 =

2 0.42 x 1000 =

3 5.7 ÷ 100 =

4 27 ÷ 1000 =

Answers

1 $361 = 361$

2 $0420 = 420$

3 $057 = 0{\cdot}057$

4 $027 = 0{\cdot}027$

Decimals

If you do not have a calculator you may need to use pen and paper methods. The same applies for aurals (mental arithmetic tests) and certain written exam questions.

Addition and subtraction of decimals

When adding or subtracting decimals remember to keep the decimal points in a straight line.

8·82 + 36 + 0·04

Remember 36 means 36·0 or 36.00

Remember to show this carry number

$$\begin{array}{r} 8\cdot82 \\ 36\cdot00 \\ 0\cdot04 \\ \hline 44\cdot86 \\ {\scriptstyle 1} \end{array}$$

Keep the decimal points in a straight line

8·3 – 2·74

add a nought

$$\begin{array}{r} 8\cdot{}^{1}3{}^{1}0 \\ 2\cdot74 \\ \hline 5\cdot56 \end{array}$$

or

add a nought

$$\begin{array}{r} {}^{7}\cancel{8}\cdot{}^{12}\cancel{3}{}^{1}0 \\ 2\cdot74 \\ \hline 5\cdot56 \end{array}$$

You can use either method. Do not forget to show the carry numbers.

Multiplication of decimals without a calculator

$$\begin{array}{r} 3.82 \\ \times\ 0.4 \\ \hline \end{array}$$

Remove the decimal points. The sum is then:

$$\begin{array}{r} 382 \\ \times\ 4 \\ \hline 1528 \end{array}$$

Count how many digits are after the decimal points in the question. There are 3 digits after the decimal points (8, 2 and 4). Therefore there will be 3 digits after the decimal point in the answer. The answer is 1.528

Division of decimals without a calculator

3.8 ÷ 0.04

We must get rid of the decimal point in the number after the ÷ sign. We move the decimal point 2 places to the right to make $0\cdot04$ into 4.

We have moved the decimal point 2 places after the ÷ sign. We must do exactly the same before the ÷ sign.

We move the decimal point 2 places to the right to make $3\cdot80$ into 380.

The sum is now 380 ÷ 4. The answer is 95.

Question

$0\cdot8 \overline{)\ 0\cdot52}$

Answer

$0\cdot8 \overline{)\ 0\cdot52} \longrightarrow 8 \overline{)\ 5\cdot20}$

Put the decimal point in the answer directly above the decimal point in the question. If required you can add noughts. The answer is 0.65.

$$8 \overline{)\ 5\cdot{}^{4}2{}^{}0} \quad 0\cdot65$$

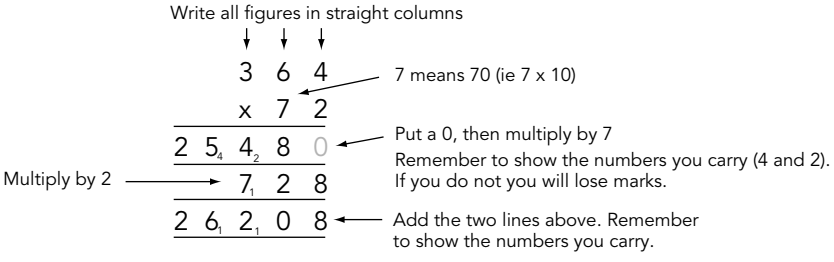
Long multiplication and division

Sometimes you are asked to work out an answer without a calculator. Show your working and you will get your marks. (Then check with a calculator.)

Non-calculator methods for long multiplication and division

You must show all of your working to prove that you have not used a calculator.

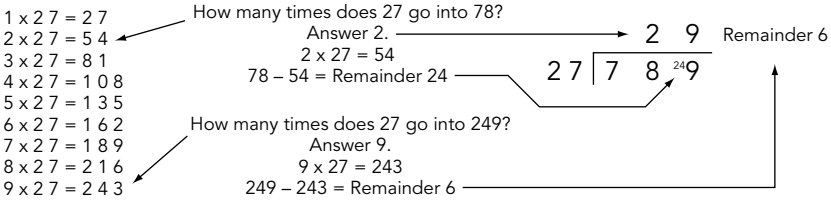
Long multiplication



The answer is 26208.

Long division

Calculate 789 ÷ 27. First write down the 27 times table.



On some calculators you can produce the 27 times table using these keys: 2 7 + +

Then keep pressing = This will produce 27, 54, 81, 108…

Questions

1 547 x 38 =

2 874 ÷ 32 =

Answers

1

	5	4	7
	x	3	8
1 6_1	4_2	1	0
4	3_3	7_5	6
2_1	0	7	8	6

2

27 Remainder 10

32 | 8 7 ^{23}4

32 times table

1 x 32 = 32
2 x 32 = 64
3 x 32 = 96
4 x 32 = 128
5 x 32 = 160
6 x 32 = 192
7 x 32 = 224
8 x 32 = 256
9 x 32 = 288

Checking

The examiner will often ask you to estimate an answer. If the question says "estimate" you must show your working. Remember that "estimate" means do not use a calculator.

Checking using inverse operations and estimating using approximation

Checking using inverse operations

Addition is the inverse (or opposite) of subtraction

Subtraction is the inverse (or opposite) of addition

Multiplication is the inverse (or opposite) of division

Division is the inverse (or opposite) of multiplication

Estimating using approximation

You can produce an approximate answer to a question by taking round estimates of the figures involved. This makes the question easy to work out without using a calculator.

Mrs Mackin bought 692 books for the Maths department at £8.95 each. She said the cost was £619.34. Without using a calculator, show how to check the answer.

Method	692 books is about 700	This method is correct also –
	£8.95 is about £9	you may have rounded £8·95 to £10
	9 x 700 = £6300	10 x 700 = £7000
	Mrs Mackin is wrong	Mrs Mackin is wrong

Whenever you check an answer, you must show your working.

Questions

1 Use inverse operations to check these answers:

a
$$\begin{array}{r} 386 \\ +\,661 \\ \hline 1047 \end{array}$$

b
$$\begin{array}{r} 582 \\ \times\,27 \\ \hline 15132 \end{array}$$

2 Estimate the cost of 395 tickets to see a pop concert. Each ticket costs £19.

Answers

1 a The inverse of addition is subtraction. 1047 – 661 = 386 The answer is correct.

b The inverse of multiplication is division. 15132 ÷ 27 = 560.444 The answer is wrong.

2 395 is about 400
£19 is about £20
400 x 20 = £8000
The cost is about £8000.

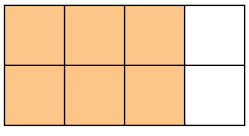
Fractions, decimals, percentages and ratio

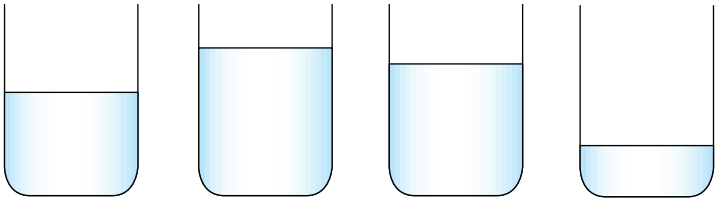
This section is a lot easier than you think. A variety of methods, including calculator methods, are shown.

Fractions and percentages

It is important to understand fractions and percentages because they are used regularly in everyday life.

$\frac{3}{4}$ means three parts out of every four parts.





this glass is $\frac{1}{2}$ full

this glass is $\frac{3}{4}$ full

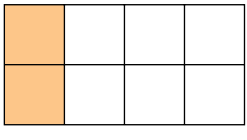
this glass is $\frac{2}{3}$ full

this glass is $\frac{1}{4}$ full

50% means $\frac{1}{2}$

25% means $\frac{1}{4}$

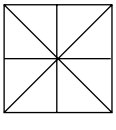
75% means $\frac{3}{4}$



25% of this shape is shaded

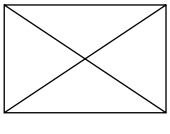
Questions

1 Shade $\frac{5}{8}$ of this shape:



2 Mark a point P which is $\frac{2}{3}$ of the way from A to B:

3 Shade 75% of this shape:

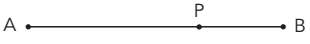


Answers

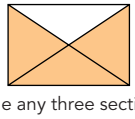
1

Shade any five sections.

2



3



Shade any three sections.

Fractions

Fractions are easy if you know how to use your calculator. Make sure your calculator has a fraction key $a\frac{b}{c}$.

Using a calculator to work out fractions

You need a calculator with a fraction key. It looks like this: $a\frac{b}{c}$

'Of' means 'Multiply'

Find $\frac{3}{8}$ of 43 means $\frac{3}{8}$ x 43

Calculator keys: [3] [$a\frac{b}{c}$] [8] [x] [4] [3] [=]

The calculator will show  This means $16\frac{1}{8}$

To calculate one number as a fraction of another number

10 people out of 25 went to work by bus. Write this as a fraction in its lowest terms.

Calculator keys: [1] [0] [$a\frac{b}{c}$] [2] [5] [=] Answer $\frac{2}{5}$

Questions

1 Find $\frac{4}{5}$ of 9

2 Find $\frac{3}{8}$ of £12

3 24 pupils out of a class of 36 passed an exam. Write this as a fraction in its lowest terms.

4 $2\frac{1}{3}$ x $\frac{4}{5}$

Answers

1 Calculator keys: [4] [$a\frac{b}{c}$] [5] [x] [9] [=]

The calculator will show  This means $7\frac{1}{5}$

2 Of means multiply $\frac{3}{8}$ x 12 = £4.50

3 Calculator keys: [2] [4] [$a\frac{b}{c}$] [3] [6] [=] Answer $\frac{2}{3}$

4 Calculator keys: [2] [$a\frac{b}{c}$] [1] [$a\frac{b}{c}$] [3] [x] [4] [$a\frac{b}{c}$] [5] [=] Answer $1\frac{13}{15}$

Changing between decimals and percentages

Percent means out of 100. Consequently dividing by 100 will change percentages to decimals, and multiplying by 100 will change decimals to percentages. This page shows you the quick way to change decimals to percentages and percentages to decimals.

Converting percentages to decimals

Move the decimal point two places to the left.

38% → 0.38 = 0·38

30% → 0.30 = 0·30 = 0·3

5% → 0.05 = 0·05

27.4% → 0.274 = 0·274

Converting decimals to percentages

Move the decimal point two places to the right.

0·52 → 52% = 52%

0·7 → 70% = 70%

0·03 → 3% = 3%

0·365 → 36.5% = 36·5%

Questions

1 Convert the following percentages to decimals:

a 74% b 6% c 42.2%

2 Change these decimals to percentages:

a 0.52 b 0.08 c 0.026

Answers

1 a 0.74 b 0.06 c 0.422

2 a 52% b 8% c 2.6%

Changing between decimals, percentages and fractions

Make sure you understand the previous page then try this page.

Converting percentages to fractions

First convert the percentage to a decimal (see previous page) and then proceed as below.

Converting decimals to fractions

$0{\cdot}3 = \frac{3}{10}$ — One number after the decimal point → One nought

$0{\cdot}37 = \frac{37}{100}$ — Two numbers after the decimal point → Two noughts

$0{\cdot}371 = \frac{371}{1000}$ — Three numbers after the decimal point → Three noughts

$0{\cdot}03 = \frac{3}{100}$ — Two numbers after the decimal point → Two noughts

Converting fractions to decimals

Divide the top number by the bottom number.

$\frac{3}{4}$	means	$3 \div 4$	$= 0.75$
$\frac{17}{20}$	means	$17 \div 20$	$= 0.85$
$\frac{3}{40}$	means	$3 \div 40$	$= 0.075$

or with a calculator:

Converting fractions to percentages

First convert the fraction to a decimal (see above), then convert the decimal to a percentage (see previous page).

Questions

1 Convert the following decimals to fractions:

a 0.4 b 0.24
c 0.02 d 0.027

2 Write these fractions as decimals:

a $\frac{3}{5}$ b $\frac{17}{25}$ c $\frac{5}{8}$

Answers

1 a $\frac{4}{10} = \frac{2}{5}$ b $\frac{24}{100} = \frac{6}{25}$
c $\frac{2}{100} = \frac{1}{50}$ d $\frac{27}{1000}$

2 a 0.6 b 0.68 c 0.625

Ratio – 1

We use ratio everyday – cooking, using maps, making drinks. You use ratio when you put sugar in coffee, eg two spoons of sugar to one cup of coffee.

Questions

1 This is a recipe for soup for four people:

800 ml water
2 tomatoes
100 g beef
8 g salt

How much of each ingredient should you use for:
a two people?
b six people?

Increase in the ratio 5:3 means multiply by $\frac{5}{3}$
Decrease in the ratio 2:7 means multiply by $\frac{2}{7}$
Increase 20 in the ratio 5:4 **Method** $20 \times \frac{5}{4} = 25$

2 Simplify these ratios:

a 4:18 b 30:45

3 The scale of a map is 1:1 000 000

a The distance between Longton and Hilton is 18 cm on the map.
What is the actual distance?

b The distance between Bursley and Higham is 142 km.
What is the distance on the map?

4 Decrease 8 in the ratio 5:16

Answers

1 a Two people will need half the ingredients: 400 ml water, 1 tomato, 50 g beef and 4 g salt
b Six people will need one and a half times the ingredients: 1200 ml water, 3 tomatoes, 150 g beef, 12 g salt

2 a 4:18, divide both sides by 2 ⟶ 2:9
b 30:45, divide both sides by 15 ⟶ 2:3

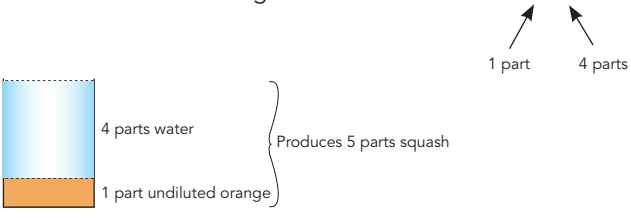
3 1:1 000 000 means 1 cm on the map represents 1 000 000 cm on the ground
1 000 000 cm = 10 000 m = 10 km
Therefore 1 cm on the map represents 10 km on the ground
a 18 cm on the map means (18 x 10) km on the ground = 180 km
b 142 km is represented by (142 ÷ 10) cm on the map = 14.2 cm

4 $8 \times \frac{5}{16} = 2{\cdot}5$

Ratio – 2

Think of ratio as an ordinary everyday bit of Maths.

You use ratio every day of your life. A simple example is making a glass of orange squash. You use undiluted orange and water in the ratio 1 : 4



How many litres of squash can be made with a three-litre bottle of undiluted orange?

The ratio is	undiluted orange		water		squash
	1	:	4	→	5
	one part		four parts		five parts

One part is 3 litres

Therefore five parts is 5 x 3 = 15 litres

Question

A man leaves £5000 in his will. The money is to be divided between his three sons Adam, Ben and Carl in the ratio 2:3:5. How much does each son receive?

Answer

Adam receives	2 parts
Ben receives	3 parts
Carl receives	5 parts
Total	**10 parts**

10 parts is £5000

Therefore 1 part is £500

Adam receives	2 parts	= £1000 (ie 2 x 500)
Ben receives	3 parts	= £1500
Carl receives	5 parts	= £2500

Percentages

This is a very important area of Mathematics.

To calculate one number as a percentage of another number you always **divide**.

Example

284 people out of 800 wore glasses. Write this as a percentage.

284 out of 800 means 284 ÷ 800

Then multiply by 100 to find the percentage,

ie 284 ÷ 800 x 100 = 35·5%

An alternative is to use the [%] key on your calculator

Calculator keys: [2] [8] [4] [÷] [8] [0] [0] [%] Answer 35.5%

Note: With some calculators you may have to press [INV] or [SHIFT] [%]

With some calculators you may have to press [=] at the end.

Questions

1 A man earns £250 per week. He receives a £4 increase.
What percentage increase is this?

2 A TV normally costs £400. In a sale this price is reduced to £340. Calculate the percentage reduction.

Answers

1 4 ÷ 250 x 100 = 1·6% or [4] [÷] [2] [5] [0] [%] = 1·6%

2 First calculate the reduction. £400 – £340 = £60
You always use the original price, ie £400
60 ÷ 400 x 100 = 15% or [6] [0] [÷] [4] [0] [0] [%] = 15%

You always **divide** if you want the answer to be %

Calculating percentage parts

This page shows you how to calculate percentages.

Using a calculator to find percentages

When you use your calculator to work out percentages, you must press the % key at the end of the calculation.

Find 18% of 6 means 18% x 6

Calculator keys: [1] [8] [x] [6] [%]

Your calculator should display 1·08. If it does not, press the [=] key

Increasing by a percentage: First find the percentage, then add.

Decreasing by a percentage: First find the percentage, then subtract.

Example

Mr James earns £270 per week. He receives an 8% increase. Calculate his new wage.

Calculator keys: [2] [7] [0] [x] [8] [%]

The calculator will show 21.6

Now add: [2] [7] [0] [+] [2] [1] [·] [6] [=]

The calculator will show 291.6 This means £291.60

Questions

1 Decrease 60 by 5%.

2 Miss Simpson saves 15% of her wages. She earns a wage of £240.
How much does she save?

Answers

1 Calculator keys: [6] [0] [x] [5] [%]

The calculator will show 3

Now subtract: [6] [0] [−] [3] [=]

The calculator will show 57 Answer 57

2 Calculator keys: [1] [5] [x] [2] [4] [0] [%] [=] Answer £36

Number patterns

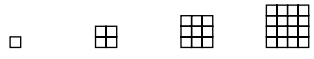
In this section you will find some patterns which may help you with your coursework projects. Learn the number patterns, eg square numbers.

Patterns you must recognise

These number patterns often appear in coursework and on examination papers. Life is much easier if you recognise them immediately.

Square numbers

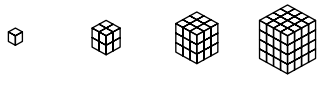
(eg 6 x 6 = 36, therefore 36 is a square number) **Note:** 6 is the square root of 36



1, 4, 9, 16, 25, 36, 49, 64, 81, 100

Cube numbers

(eg 5 x 5 x 5 = 125, therefore 125 is a cube number) **Note:** 5 is the cube root of 125



1, 8, 27, 64, 125, 216, 343, 512, 729, 1000

Triangle numbers

1, 3, 6, 10, 15, 21, 28, 36, 45, 55

(1) (1+2) (1+2+3) (1+2+3+4) etc

Fibonacci sequence

(Add the two previous terms in the sequence)

1, 1, 2, 3, 5, 8, 13, 21, 34, 55

(1+1=2) (1+2=3) (2+3=5) (3+5=8) (5+8=13) etc

Information you should know

Multiples: The multiples of 3 are 3, 6, 9, 12, 15…
Any number in the 3 times table is a multiple of 3, eg 36, 42, 300.

Factors: The factors of 12 are 1, 2, 3, 4, 6, 12
Any number which divides exactly into 12 is a factor of 12.

Prime numbers: Prime numbers have **exactly two** factors.
The prime numbers are 2, 3, 5, 7, 11, 13, 17, 19…

Note: 1 is not a prime number because it has only one factor.

Formulae

This is algebra. That's just Maths with letters instead of numbers. It looks harder but the rules are just the same. Spend some time on the example below. Replacing letters with numbers helps your brain understand what is being done and is the 'secret' to algebra. **If you can't do it with letters replace the letters with numbers**. Use easy numbers and avoid using 0 or 1 – strange things happen if you do.

Using formulae

You need to be able to use simple formulae. You will replace words with numbers.

This is a formula

Telephone bill = Rental charge + Cost of telephone calls

The telephone bill is calculated by adding the rental charge and the cost of the calls.

This is an instruction

Choose a number, double it, then add 3 to your answer.

If you choose the number 8:	Start number,	double it,	add 3
	8	16	19

Questions

1 Use the telephone bill formula.

a What is the bill if the rental charge is £12 and the cost of the calls is £41?

b What is the cost of the telephone calls if the rental charge is £12 and the telephone bill is £76?

2 Use the number instructions above with the following start numbers:

a 7 b 12 c 0

d Find the start number if the result is 37.

Answers

1 a Telephone bill = Rental charge + Cost of telephone calls
Telephone bill = £12 + £41 = £53

b Telephone bill = Rental charge + Cost of telephone calls
£76 = £12 + ?
Cost of telephone calls = £64

2

	start number	double it	add 3	**Answer**
a	7	14	17	**17**
b	12	24	27	**27**
c	0	0	3	**3**

d To find the start number, the formula must be reversed:
Work the formula backwards. Instead of adding 3, we take away 3. 37 – 3 = 34
Instead of doubling, we halve the number. half of 34 = 17
17 is the start number

Writing in algebra

Example

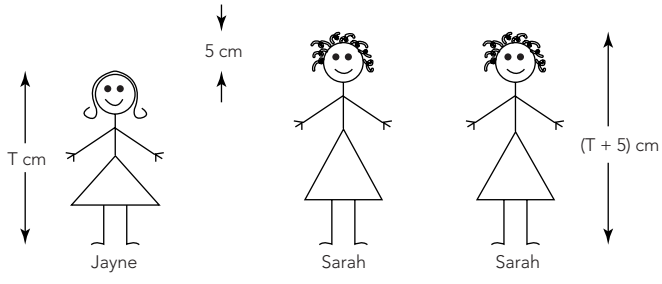
Sarah is 5 cm taller than Jayne. Jayne is T cm tall. How tall is Sarah?

If you do not know how to write the answer using symbols, try using numbers instead of the letters,

eg Suppose Jayne is 100 cm tall, Sarah is 100 + 5
Suppose Jayne is 120 cm tall, Sarah is 120 + 5
Suppose Jayne is 140 cm tall, Sarah is 140 + 5

Now try letters.

Suppose Jayne is T cm tall, Sarah is (T + 5) cm tall



Questions

1 A boy has X pence. He spends Y pence. How much does he have left?

2 A man buys P oranges at Q pence each. What is the total cost?

Answers

1 Try using numbers instead of letters
Suppose the boy has 30 pence and spends 10 pence 30 – 10
Suppose the boy has 40 pence and spends 25 pence 40 – 25
Suppose the boy has 80 pence and spends 30 pence 80 – 30
Now try letters
Suppose the boy has X pence and spends Y pence X – Y

2 Try using numbers instead of letters
Suppose the man buys 4 oranges at 7 pence each 4 x 7
Suppose the man buys 6 oranges at 8 pence each 6 x 8
Suppose the man buys 9 oranges at 3 pence each 9 x 3
Now try letters
Suppose the man buys P oranges at Q pence each P x Q

Using algebra

The more you do algebra, the easier it becomes. The examples below show you the types of questions you can expect to meet.

Examples

Given a = 2, b = 3, c = 4, work out the following:

1 ab 2 abc 3 $\frac{1}{4}$ab

4 (a+b)c 5 a(b+c)

Method

1 ab means a x b

2 x 3

Answer 6

2 abc means a x b x c

2 x 3 x 4

Answer 24

3 $\frac{1}{4}$ab means $\frac{1}{4}$ x a x b

$\frac{1}{4}$ x 2 x 3

Answer 1·5

4 (a+b)c means (a + b) x c

(2 + 3) x 4

Calculator keys:

(2 + 3) x 4 =

Answer 20

5 a(b+c) means a x (b + c)

2 x (3 + 4)

Calculator keys:

2 x (3 + 4) =

Answer 14

Questions

1 $A = \frac{1}{2}BH$ Find A when B = 8 and H = 4

2 P = 2(L+W) Find P when L = 6 and W = 4

3 $D = \frac{A+E}{BC}$ Find D when A = 20, B = 2, C = 5 and E = 40

Answers

1 A = $\frac{1}{2}$ x B x H

= $\frac{1}{2}$ x 8 x 4

= 16

2 P = 2 x (L + W)

= 2 x (6 + 4)

Calculator keys:

2 x (6 + 4) =

= 20

3 $D = \frac{A+E}{BC}$

$= \frac{(20 + 40)}{(2 \times 5)}$

Calculator keys: (2 0 + 4 0) ÷ (2 x 5) =

Answer = 6

Equations

I have given you a few basic rules and then some examples to show how they work.

Rules

You must know all of these rules. When you do the questions make sure your working is the same as shown in the answers. Don't just say "I can see the answer, I don't need to do any working". Try to get into good habits. You will not be able to just "see the answer" when numbers are difficult.

+ is the opposite of –	– is the opposite of +
x is the opposite of ÷	÷ is the opposite of x

Rules for solving equations

1 3a means 3 x a

2 The sign in front of a number is attached to that number,
eg -3 + 6a. The – is attached to the 3, the + is attached to 6a.

3 Always keep the equals signs in straight columns. Work down the page not across.

4 When you take a number from one side of the equals to the other:

+ becomes –	– becomes +
x becomes ÷	÷ becomes x

5 Do the addition and subtraction parts before the multiplication and division.

6 Letters one side, numbers the other (see question 7).

Questions

1 $a + 5 = 8$ 2 $a - 2 = -7$ 3 $-7y = 28$ 4 $\frac{y}{3} = 6$

5 $5a + 7 = 27$ 6 $^{a}/_{3} - 5 = 1$ 7 $8a + 6 = 5a - 21$

Answers

1 $a + 5 = 8$
$a = 8 - 5$
$a = 3$

2 $a - 2 = -7$
$a = -7+2$
$a = -5$

3 $-7y = 28$
$y = {}^{28}/_{-7}$
$y = -4$

4 $^{y}/_{3} = 6$
$y = 6 \times 3$
$y = 18$

5 $5a + 7 = 27$ Deal with the add first
$5a = 27 - 7$
$5a = 20$ Now deal with the multiplication
$a = {}^{20}/_{5}$
$a = 4$

6 $^{a}/_{3} - 5 = 1$
$^{a}/_{3} = 1 + 5$
$^{a}/_{3} = 6$
$a = 6 \times 3$
$a = 18$

Keep equals signs in straight columns

7 $8a + 6 = 5a - 21$
$8a - 5a = -21 - 6$
$3a = -27$
$a = {}^{-27}/_{3}$
$a = -9$

Writing equations

Remember "putting numbers in for letters" helps your brain to understand.

Advice: Look at page 21 *Writing in algebra* before you try this page.

Formulating equations

You must understand a problem before you can write an equation to solve it. Try putting numbers in for the letters. This will help you to understand what the question is asking.

Questions

1 A man buys t apples at 8p each. The total cost is 96p.

a Form an equation to show this
b Solve the equation

2 I think of a number N, I double it and add 15. The answer is 31.

a Form an equation to show this
b Solve the equation

Answers

1 a Try putting numbers in for the letters.

5 apples = 8 x 5 = 40
6 apples = 8 x 6 = 48
7 apples = 8 x 7 = 56
t apples = 8 x t = 96

The equation is 8t = 96

b 8t = 96

$t = \frac{96}{8}$

t = 12

2 a Choose numbers. See what happens:

if N = 3 3 x 2 + 15 = 21
if N = 4 4 x 2 + 15 = 23
if N = 5 5 x 2 + 15 = 25
Try N N x 2 + 15 = 31

The equation is N x 2 + 15= 31 or 2N + 15 = 31

b N x 2 + 15 = 31
N x 2 = 31 – 15
N x 2 = 16

$N = \frac{16}{2}$

N = 8

Trial and improvement

This used to be called trial and error. But mathematicians do not like errors so they changed the name to improvement. Make sure you remember the four columns.

WARNING: This topic can be a time-waster in the examination. If you are short of time this is a question to leave and go back to at the end.

Trial and improvement

You should draw four columns as shown below.

In the first column write down your guess.
In the second column work out the answer using your guess.

If your answer is too big write your **guess** in the 'too big' column.
If your answer is too small write your **guess** in the 'too small' column.

Guess x	Answer	Too big	Too small

Question

$x^3 = 151$

Find the value of x correct to one decimal place using trial and improvement methods.

Answer

You must show your working. For example, start by guessing 5. You may have used different guesses in your calculations.

Guess x	Answer x^3	Too big	Too small
5	5 x 5 x 5 = 125		5
6	6 x 6 x 6 = 216	6	
5·5	5·5 x 5·5 x 5·5 = 166·375	5·5	
5·3	5·3 x 5·3 x 5·3 = 148·877		5·3
5·4	5·4 x 5·4 x 5·4 = 157·464	5·4	
5·35	5·35 x 5·35 x 5·35 = 153·130	5·35	

5 is too small. Guess higher.

5 is too small. 6 is too big. Guess between 5 and 6.

5 is too small. 5·5 is too big. Guess between 5 and 5·5.

5·3 is too small. 5·5 is too big. Guess between 5·3 and 5·5.

The answer to one decimal place is either 5·3 or 5·4. To find out which try the number in the middle of 5·3 and 5·4, ie 5·35.

5·35 is too **big** so 5·3 is **nearer**

Answer = 5·3

Graphs

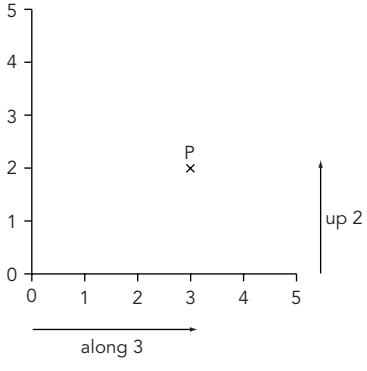
Graphs must be neat and tidy or you will lose marks.

Co-ordinates

Co-ordinates are used to identify points on a graph.

P is the point (3,2)

Start from 0.
Go along 3, then go up 2.



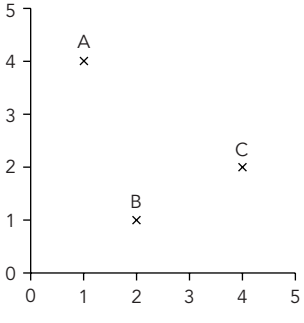
Questions

1 Give the co-ordinates of the following points:

A (,)

B (,)

C (,)



2 Mark these points on the graph:

X is the point (3,2)

Y is the point (5,4)

Z is the point (1,3)

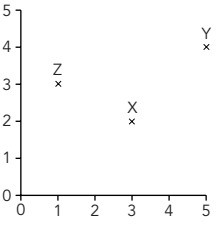
Answers

1 A (1,4)

B (2,1)

C (4,2)

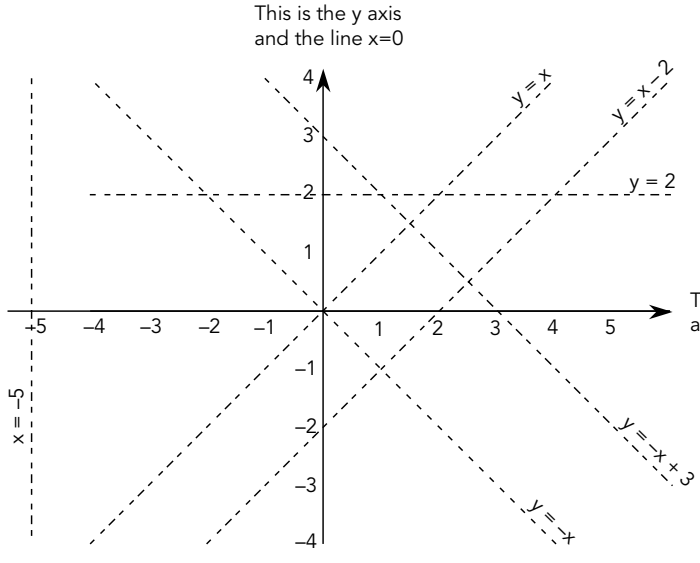
2



Drawing lines

Algebra and graphs are closely connected. You must be able to illustrate algebraic information in graphical form.

Graphical representation



You must be able to draw the lines y = x and y = –x

Question

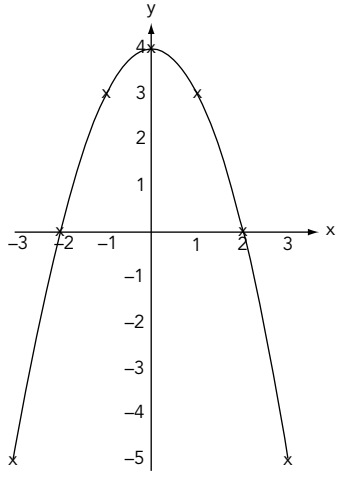
Complete this table of values and draw the graph of $y = -x^2 + 4$

x	–3	–2	–1	0	1	2	3
y							

Answer

When x = –3 $y = -(-3)^2 + 4 = -5$

x	–3	–2	–1	0	1	2	3
y	–5	0	3	4	3	0	–5



Angles

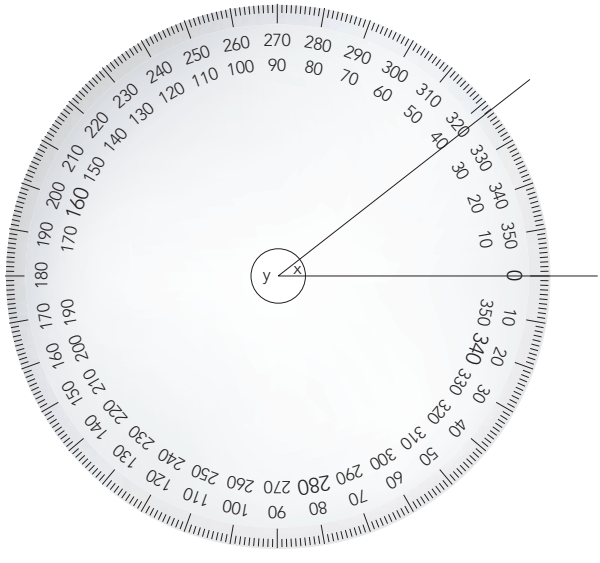
Algebra is finished. Actually it gets easier from now on and we are nearly halfway through. Just make sure you can use a protractor (I advise a 360° one) and you know the rules.

Using a protractor

Protractors are used to measure angles. A small circular protractor, diameter 10 cm is the easiest to use.

Measuring and drawing angles

This is a 360° protractor. It is used to measure angles. It is easier to measure large angles with a 360° protractor, than with a semi-circular protractor.



The centre of the protractor must be placed on the point where the two lines meet. You must read the angle very carefully. The angle must be exact. If you are more than 1° away from the exact answer, it is wrong.

Place 0° on the protractor on the line.

Question

Use the protractor drawing above to measure: a angle x and b angle y.

Answer

a Look at the protractor. There are two numbers at each point. It is obvious that angle x is less than 180°.
Note: A common mistake is to read the angle as 42°. This is wrong. The angle is between 30° and 40°.
Angle x = 38°.

b It is obvious that angle y is more than 180°. Angle y = 322°.

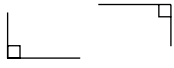
Angles: Acute, obtuse, reflex

You need to know the special names for angles.

Using language associated with angles

An **acute** angle is less than 90°.
These angles are acute.

A **right** angle is 90°.
These are right angles.

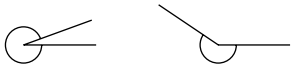


We show a right angle with a box in the corner.

An **obtuse** angle is between 90° and 180°.
These angles are obtuse.

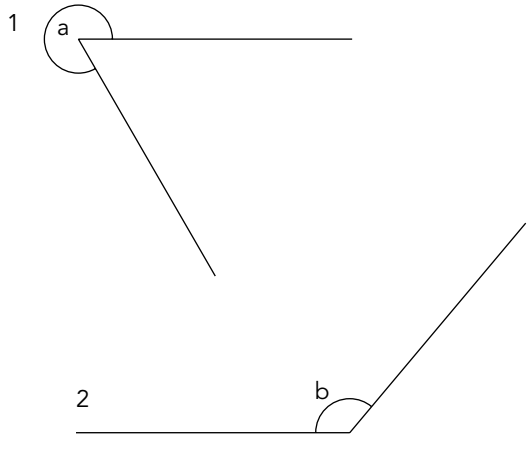
There are 180° on a straight line.

A **reflex** angle is between 180° and 360°.
These angles are reflex.



Question

Measure and name these angles:



Answers:

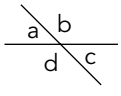
1 a is a reflex angle 300°

2 b is an obtuse angle 130°

Intersecting and parallel lines

You need to know the following information about angles. You often need to extend lines to make Z shapes. If you are used to seeing parallel lines going across the page and a question has the lines going down the page it can sometimes help to turn the paper around.

Intersecting lines



$a + b = 180°$
$b + c = 180°$
$c + d = 180°$
$d + a = 180°$

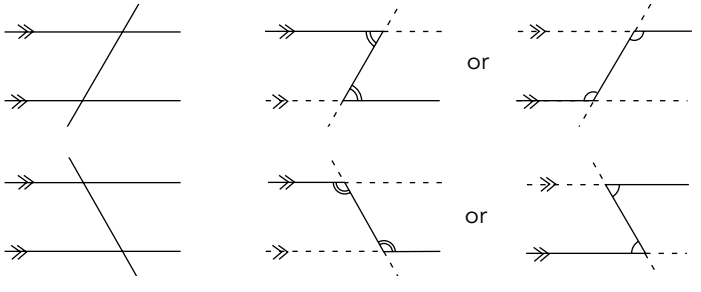
} Angles on a straight line add up to 180°

$a = c$
$b = d$

} Vertically opposite angles are equal

Parallel lines

Look for Z shapes. Angles at corners of Z shapes are equal.

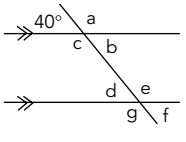


Angles at the corners of Z shapes are called **alternate angles**.

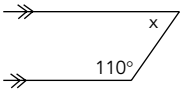
These angles are called **corresponding angles.**

Questions

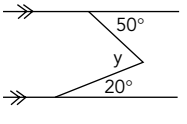
1 Find the missing angles:



2 Find x:



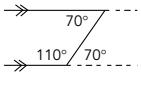
3 Find y:



Answers

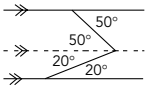
1 a = 140°, b = 40°, c = 140°, d = 40°, e = 140°, f = 40°, g = 140°.

2 It often helps to extend the parallel lines to produce Z shapes.



x = 70°

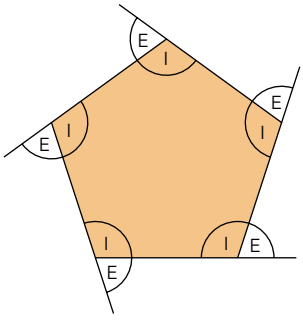
3 Try adding an extra parallel line



y = 70°

Regular polygons

A polygon is a shape made from straight lines. A regular polygon has all of its sides the same length and all of its angles the same size.



I = Interior angles
E = Exterior angles

> The sum of the exterior angles of a polygon is 360°
> Interior angle + exterior angle = 180°

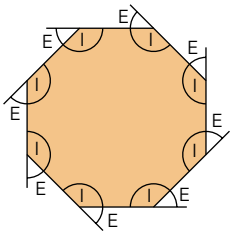
Questions

1 Find the size of an exterior and an interior angle of a regular octagon.

2 Find the size of an exterior and an interior angle of a regular hexagon.

Answers

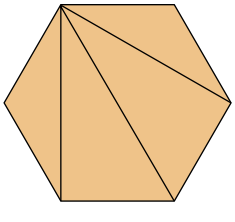
1



An octagon has 8 sides, 8 exterior angles, 8 interior angles

8 exterior angles°	= 360°
Therefore 1 exterior angle	$= \frac{360}{8} = 45°$
Interior angle + exterior angle	= 180°
Interior angle + 45°	= 180°
Interior angle	= 135°

2 This question can be solved using the above method.
An alternative method is to split the shape into triangles.



4 triangles are formed
Therefore the sum of the interior angles is

4 x 180°	= 720°
6 interior angles	= 720°
1 interior angle	= 120°
Interior + exterior	= 180°
120° + exterior	= 180°
Exterior	= 60°

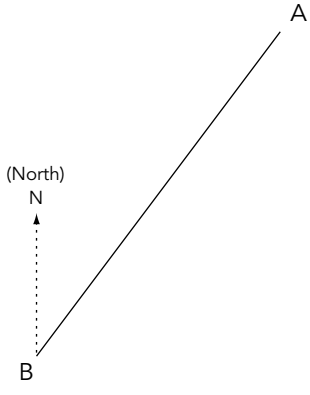
Bearings

Bearings are measured clockwise from North. They are easier to measure with a circular protractor (diameter 10 cm). North will usually be shown as vertically up the page. Ensure the 0° on the protractor is on the North line. REMEMBER if the question states "measure the bearing of C from D" you put your protractor on D. Put your protractor on the "from" part of the question.

Questions

1 What is the bearing of A from B?

2 What is the bearing of B from A?

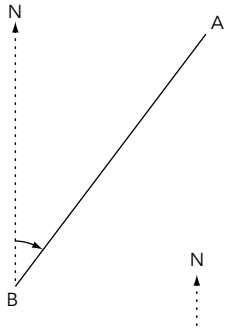


Note: If you know the bearing from A to B, then the bearing from B to A will be 180° more or 180° less, eg:

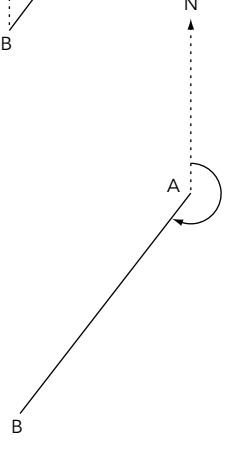
037° + 180° = 217°
217° – 180° = 037°

Answers

1 Bearings are always measured clockwise from North.
Place your protractor on B.
Measure the angle between north and AB.
The angle is 37°.
Bearings are always written as three figures. Answer = 037°



2 Place your protractor on A.
Measure the angle.
The angle is 217°.
Answer 217°



Most of this section is straightforward. Learn the rules.

Common 2-D shapes

You need to be able to recognise 2-D shapes in various positions. When attempting the last two questions on this page it may help to trace the shape on tracing paper and then turn the tracing paper to find where the new shape is formed.

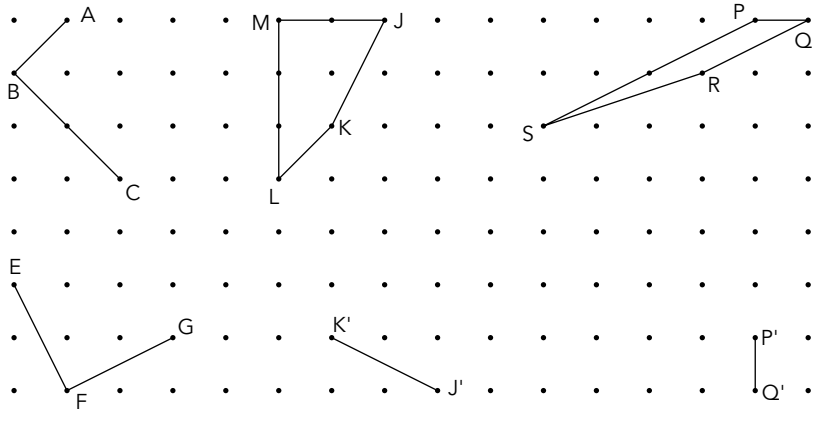
Questions

1 ABCD is a rectangle. Complete the diagram.

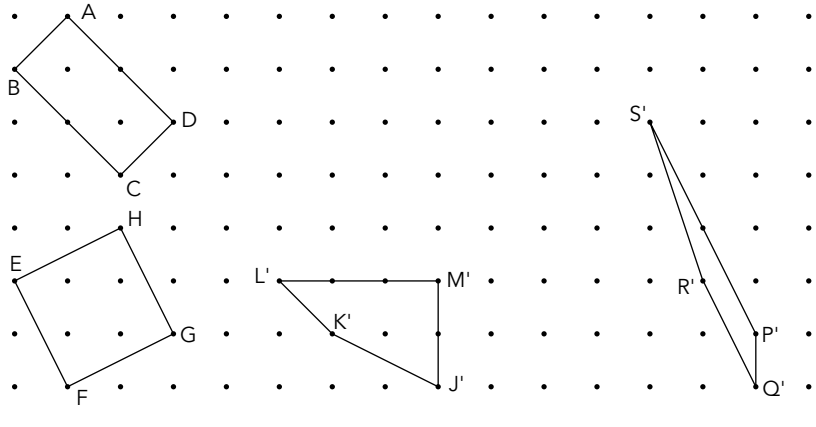
2 EFGH is a square. Complete the diagram.

3 The shape JKLM is turned and moved. J'K' are shown. Complete the diagram and mark the points L'M'.

4 The shape PQRS is turned and moved. P'Q' are shown. Complete the diagram and mark the points R'S'.



Answers

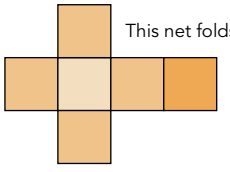


2-D representations of 3-D shapes

Most of this is common sense. If you are asked to make an accurate drawing, the drawing must be correct, if any length is more than 1 mm out you will lose marks.

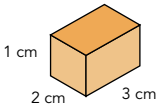
2–D representations of 3–D shapes

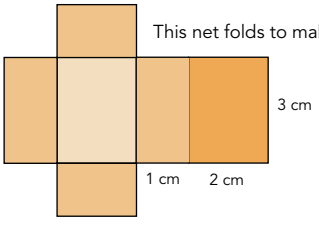
Cube



This net folds to make a cube

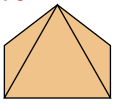
Cuboid

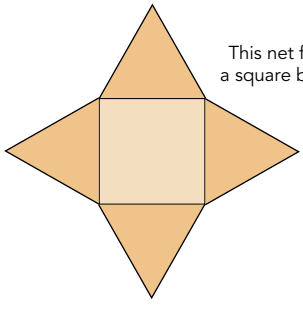




This net folds to make a cuboid

Square-based pyramid





This net folds to make a square based pyramid

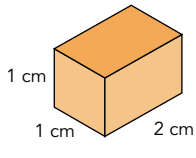
You also need to know the names of these 3-D shapes:

Tetrahedron or **triangular-based pyramid**

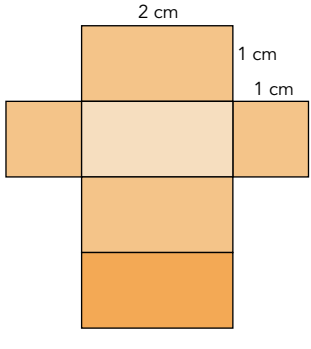
Triangular prism

Question

Draw an accurate 2-D net of this cuboid.



Answer

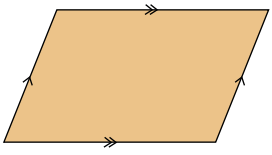


Properties of quadrilaterals

No shortcuts here. You are expected to know these properties. You will have to learn them.

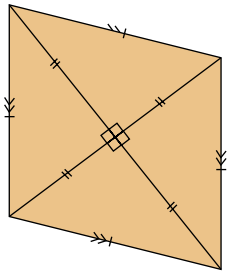
A quadrilateral is a four-sided shape. The angles add up to 360°. You are expected to know the following information about these quadrilaterals.

Parallelogram



Opposite sides are parallel and the same length.
Opposite angles are equal.
Diagonals bisect each other.
Rotational symmetry order 2.

Rhombus

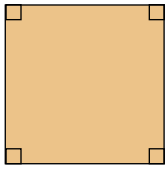


This is a parallelogram with four equal sides.
Diagonals bisect each other.
Rotational symmetry order 2.

Rectangle

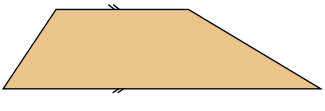
A parallelogram with all angles equal (ie 90°).
Rotational symmetry order 2.

Square



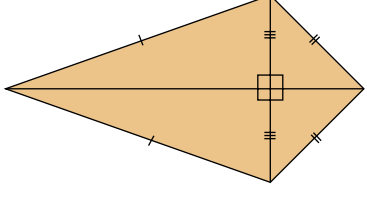
A rectangle with all sides equal length.
Rotational symmetry order 4.

Trapezium



A quadrilateral with one pair of parallel sides.
No rotational symmetry.

Kite

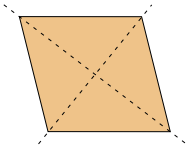


Two pairs of equal length sides adjacent to each other.
Diagonals cross at right angles.
One diagonal bisects the other.
No rotational symmetry.

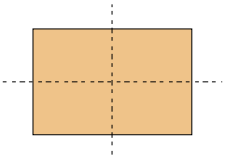
Properties of quadrilaterals and triangles

These diagrams show the axes of symmetry.

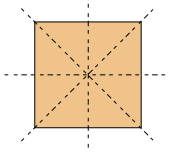
Rhombus



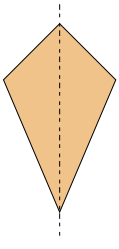
Rectangle



Square



Kite



Parallelogram

Usually none.

Trapezium

Usually none.

You also need to know the special names of these two triangles:

Equilateral

An equilateral triangle has three axes of symmetry. If you fold on any axis of symmetry you produce two identical right-angled triangles.

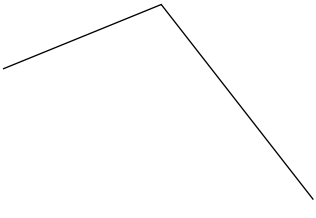
Isosceles

Two sides equal. Two angles equal.

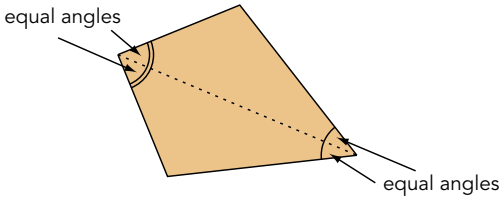
An isosceles triangle has one axis of symmetry. If you fold on the axis of symmetry you produce two identical right-angled triangles.

Question

This drawing shows two sides of a kite. Complete the drawing.

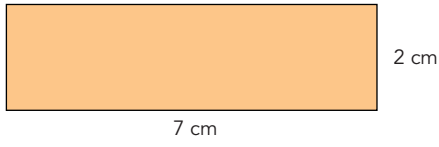


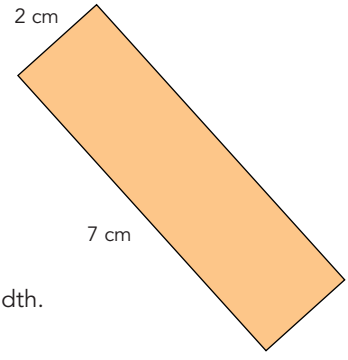
Answer



Congruent shapes

If two shapes are identical then they are congruent.
Look at these two rectangles:

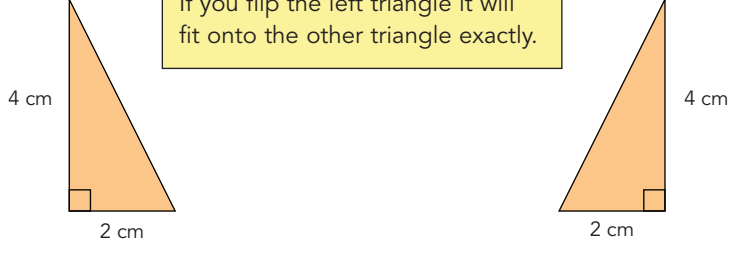




The rectangles have the same length and breadth.
Therefore the shapes are congruent.

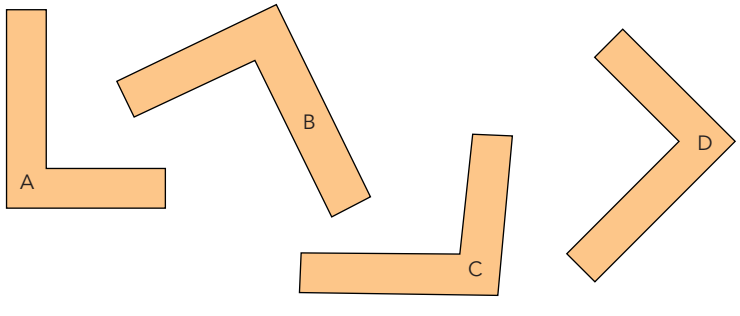
Shapes are congruent if one shape can be cut out and fitted exactly onto the other shape. An easy way to check is to use tracing paper. Trace the first shape, and see if it will fit exactly onto the other shape. You can turn the shape or flip the shape.

These triangles are congruent.
If you flip the left triangle it will fit onto the other triangle exactly.



Question

Here are four shapes. Which two are congruent?



Answer

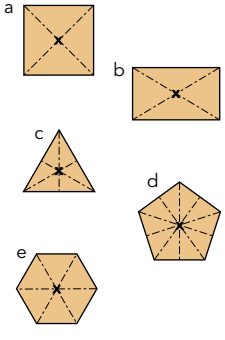
A and D are congruent. (If you trace shape A you will find that it fits onto shape D exactly.)

Symmetry

There are two types of symmetry: rotational symmetry produced by turning, and symmetry produced by folding to make mirror images.

Rotational symmetry

A shape has **rotational symmetry** if it fits exactly onto its original outline more than once in a complete turn. The number of times that it fits is called the order of rotational symmetry. If a shape only fits onto itself once we say that it has no rotational symmetry, or that it has order 1.



Shape	Name	Order of rotational symmetry
a	Square	4
b	Rectangle	2
c	Equilateral triangle	3
d	Regular pentagon	5
e	Regular hexagon	6

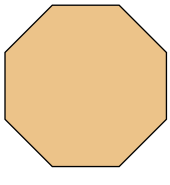
X marks the centre of rotation.

To find the centre of rotation: If a shape has an even number of sides, join opposite corners. If a shape has an odd number of sides, join each corner to the centre of the opposite side.

Question

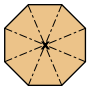
What is the order of rotational symmetry of this regular octagon?

Mark the centre of rotational symmetry.



Answer

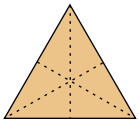
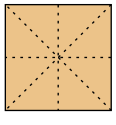
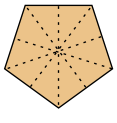
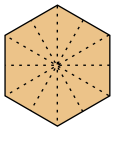
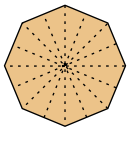
Make a tracing of the octagon. Turn the tracing through one complete turn, ie 360°. It fits the original octagon exactly 8 times. Therefore the order of rotational symmetry is 8.



Symmetry of 2-D shapes – 1

Axes of symmetry

If you fold a shape along an axis of symmetry, the shape will fit exactly onto itself. If the shape is a regular shape, all sides are equal and all angles are equal. Here are some regular shapes – the dotted lines indicate axes of symmetry.

	Name	Number of sides	Axes of symmetry
	Equilateral triangle	3	3
	Square	4	4
	Regular pentagon	5	5
	Regular hexagon	6	6
	Regular octagon	8	8

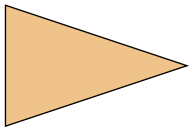
Note: If the shape is regular

Number of sides = Number of axes of symmetry

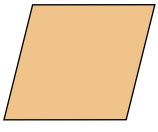
Symmetry of 2-D shapes – 2

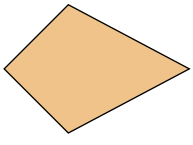
Questions

Here are some common 2–D shapes. Use dotted lines to indicate axes of symmetry. Give the names of the shapes if you can.

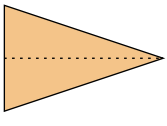
1 

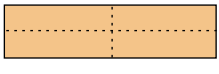
2 

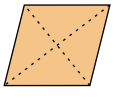
3 

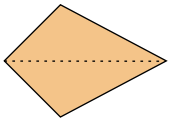
4 

Answers

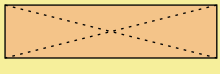
1  Isosceles triangle

2  Rectangle

3  Rhombus

4  Kite

A common error is



These are **not** axes of symmetry.

Note: There is only one axis of symmetry on the kite.

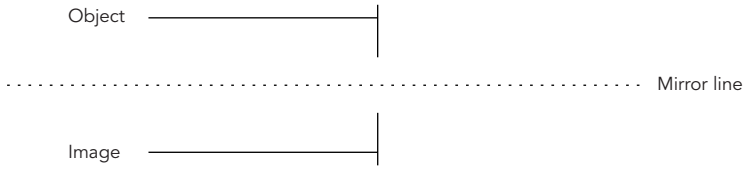
Transformations

The main types of transformation are:

- **Translation** – slide the shape left, right, up or down.
- **Rotation** – turn it around.
- **Reflection** – mirror image, ie turn it over.
- **Enlargement** – make it bigger (or smaller).

Reflection

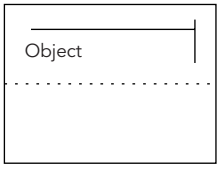
The image you see in a mirror is a **reflection**. The image is exactly the same but it has been flipped over. **Remember:** you can ask for tracing paper in the exam.



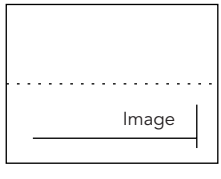
Note: The object and the image are equal distances from the mirror line.

How to use tracing paper to reflect an object

Trace the object and the mirror line onto tracing paper.



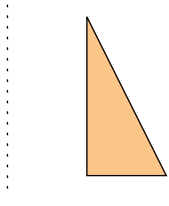
Flip the tracing paper.



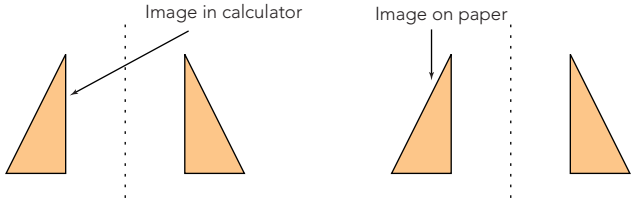
Place the tracing of the mirror line directly on top of the mirror line on the paper. This will show where the reflection image should be.

Question

Draw the reflection of the triangle.



Answer



If you do not have a mirror, check your answer using the display on your calculator. Tilt the calculator towards the object and place on the mirror line (the same way you would use a mirror). Look into the answer display to see where the image should be. Remove the calculator and check your drawn image is in the same place.

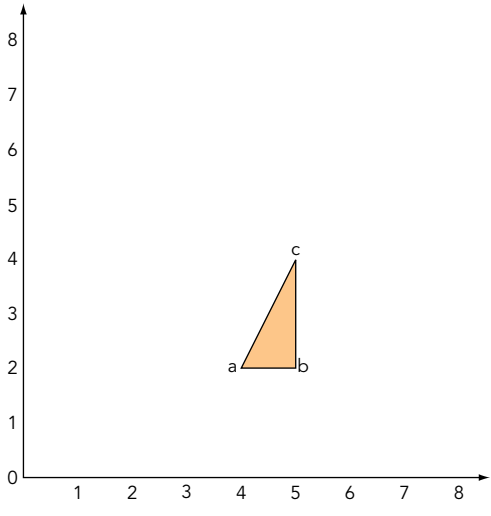
Enlargement – 1

Enlargement means making bigger or smaller. There are several methods of enlarging. The advantage of the method shown is that it can be used for enlargement by a whole number scale factor and a fractional scale factor.

Questions

1 Enlarge the triangle ABC by a scale factor of 2.

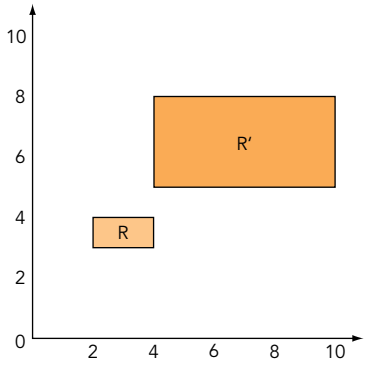
Centre of this enlargement is the point (2,1).



2 R′ is an enlargement of R.

a What are the co-ordinates of the centre of enlargement?

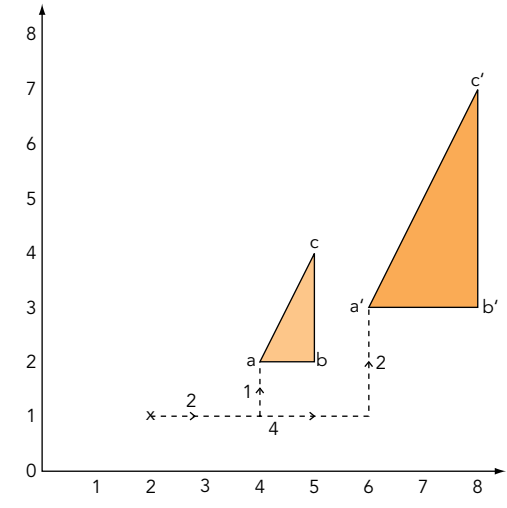
b What is the scale factor of the enlargement?



Enlargement – 2

Answers

1



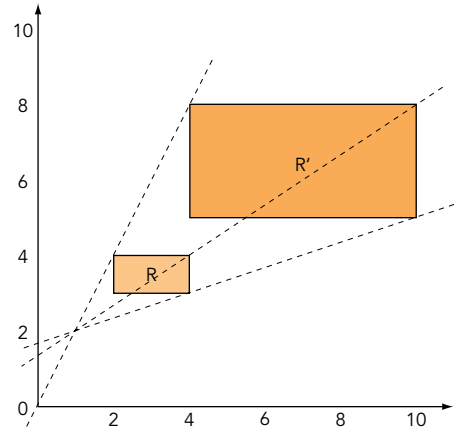
Count the distance from the centre of enlargement to each point

scale factor

Point a		**Point a'**
1 up / 2 along	x 2	2 up / 4 along
Point b		**Point b'**
1 up / 3 along	x 2	2 up / 6 along
Point c		**Point c'**
3 up / 3 along	x 2	6 up / 6 along

Note: Always count from the centre of enlargement.

2



a Use a ruler to join the corners.
The dotted lines cross at (1,2).
Therefore the centre of enlargement is the point (1,2)

To find the scale factor, you must measure the length of any side of R' (ie the new length) and the corresponding length of R (ie the original length).

Example:
The top side of R' has a length of 6
The top side of R has a length of 2

b Scale factor = $\frac{\text{new length}}{\text{original length}}$

Scale factor = $\frac{6}{2} = 3$

Measurement

Quite a lot to memorise if you don't already know it. Most of this section is everyday Maths. Nothing difficult.

Metric units of measure

You need to know the following information about metric units.

Length

We use millimetres, centimetres, metres and kilometres.

This is a millimetre (1 mm): –

This is a centimetre (1 cm): ———

A metre (1 m) is about one large pace.

A kilometre (1 km) is about one thousand large paces.

It takes about ten minutes to walk one kilometre.

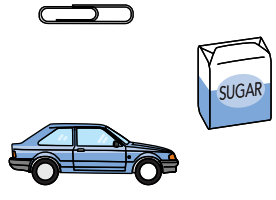
Mass

We use grams, kilograms and tonnes.

A paper-clip has a mass of about one gram (1 g).

A bag of sugar has a mass of about one kilogram (1 kg).

A small car has a mass of about one tonne (1 t).



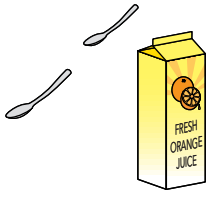
Capacity

We use millilitres, centilitres and litres.

A small spoon for medicine has a capacity of 5 millilitres (5 ml).

A teaspoon holds about one centilitre (1 cl).

An orange juice container holds about 1 litre (1 l).



Questions

1 What units would you use to measure the length of this piece of paper?

Choose from millimetres, centimetres, metres, kilometres.

2 What is the missing number in the following sentence?

A bag of crisps has a mass of ..... grams. Choose from 5, 25, 500, 1000.

Answers

1 Centimetres would be the most appropriate units. Millimetres would be acceptable.

2 25

Rough metric equivalents of Imperial units

Imperial units are the old units we used to use before the metric system. Your parents probably still use feet, inches, pounds and stones. Until people stop using Imperial units you have to understand both.

You should know the following information:

	Imperial units
Units of length 1 inch is about 2.5 centimetres 1 foot is about 30 centimetres 1 yard is about 1 metre 5 miles are about 8 kilometres	12 inches = 1 foot 3 feet = 1 yard
Units of mass 1 ounce is about 30 grams 2 pounds are about 1 kilogram	16 ounces = 1 pound 14 pounds = 1 stone
Units of capacity 1 pint is about 0.5 litre 1 gallon is about 4.5 litres	8 pints = 1 gallon

Questions

1 My pencil is 8 inches long. How many centimetres is this?

2 My car's petrol tank holds 10 gallons. How many litres is this?

3 A newborn baby weighs 7 pounds. How many ounces is this?

Answers

The following are approximate.

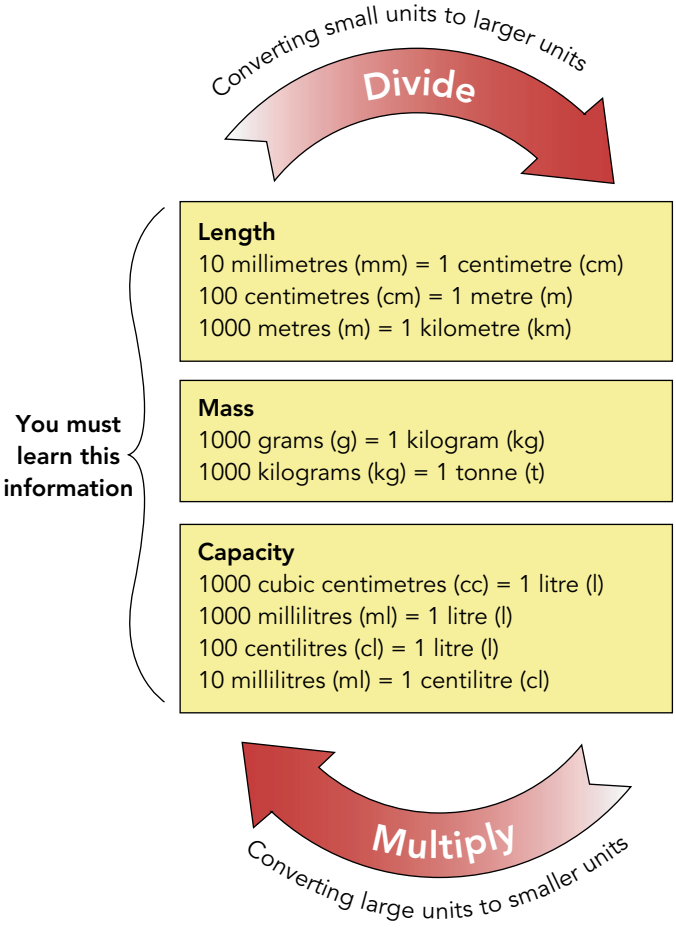
1 8 x 2.5 = 20 centimetres

2 10 x 4.5 = 45 litres

3 7 x 16 = 112 ounces

Converting one metric unit to another

Metric units are easy to convert, you always multiply or divide by 10, 100 or 1000.
Look back at page 7 *Mental arithmetic shortcuts – 2*.



Questions

1 Convert 524 centimetres into metres

2 Convert 3.56 tonnes into kilograms

Note: 1 cm^3 = 1 cc = 1 ml

Answers

1 524 ÷ 100 = 5.24 metres

2 3.56 x 1000 = 3560 kilograms

Making sensible estimates

Estimate simply means guess, but your guess has to be sensible. Often you will have to estimate because you do not have enough time to work out the exact answer.

You will often be asked to make sensible estimates of length, mass and capacity. The easiest way to do this is to know the length, mass and capacity of some common objects.

Guess the answers to the following and write your guess in pencil. Then measure the exact answers and write them in the box.

	Guess	Exact
Length		
1	My finger is _____ centimetres long.	
2	My pace is _____ centimetres long.	
3	My classroom is _____ metres long.	
4	1 kilometre is the distance from _____ to _____ .	
Mass		
5	My pencil weighs _____ grams.	
6	I weigh _____ kilograms.	
7	A Mini car weighs _____ tonne.	
Capacity		
8	A can of cola holds _____ millilitres.	
9	A giant bottle of cola holds _____ litres.	

Answers

(Your answers will probably be within these ranges.)

1 5–10 centimetres
2 70–100 centimetres
3 3–8 metres
4 Try walking for 10 minutes. This distance will be about 1 kilometre.
5 5–10 grams
6 40–100 kilograms
7 1 tonne
8 Approximately 330 millilitres – read the label to check
9 Probably 3 litres – read the label to check

You will need to give units to a sensible degree of accuracy. Try these:

Questions

1 Which unit would you use to measure the distance from London to Manchester?

2 The speed of a car is 76·8327 km/h. Write this to a sensible degree of accuracy.

Answers

1 kilometres
2 77 km/h or 76·8 km/h

Time

Your examination paper will contain a time question. You must learn the units of time.

Remember: there are 60 minutes in an hour, not 10 or 100.

Questions

1 A woman took 2 hours 17 minutes to travel to work. Write this in minutes.

2 A train left Glasgow at 07:42 and arrived in London at 13:16. How long did the journey take?

3 A ferry left Poole at 22:38 on Tuesday and arrived in Cherbourg at 07:21 on Wednesday. How long did the journey take?

Memorise:

60 seconds = 1 minute

60 minutes = 1 hour

24 hours = 1 day

365 days = 1 year

Answers

1 There are 60 minutes in each hour.

2 hours = 120 minutes

2 hours 17 minutes = 137 minutes

2 If you have 5 hours 34 minutes, continue to use your own method. If not, you must look at the solution.

Common errors are 5 hours 74 minutes or 6 hours 14 minutes

First		**Hours**	**Minutes**
Find the time to the next whole hour	07:42 – 08:00		18
Then the hours	08:00 – 13:00	5	
Then the minutes	13:00 – 13:16		16
		5 hours	34 minutes

3 Same method as question 2.

A **common error** is 15 hours 17 minutes

First		**Hours**	**Minutes**
Find the time to the next whole hour	22:38 – 23:00		22
Then the hours	23:00 – 07:00	8	
Then the minutes	07:00 – 07:21		21
		8 hours	43 minutes

Perimeter, area and volume

You need to know the meaning of and units used to measure perimeter, area and volume.

Perimeter, area and volume

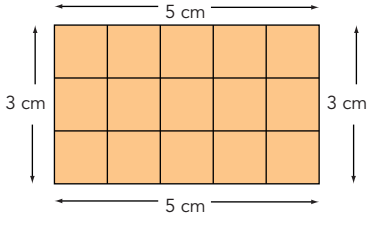
Perimeter

The perimeter of a shape is the distance around the shape.

3 cm + 5 cm + 3 cm + 5 cm = 16 cm

Area (always measured in units^2, eg mm^2, cm^2, m^2)

To find the area, count the number of squares. There are 15 squares. Each square has an area of 1 cm^2. The area is 15 cm^2.



or length x width

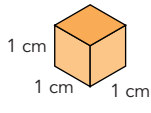
5 cm x 3 cm = 15 cm^2

Volume (always measured in units^3, eg mm^3, cm^3, m^3)

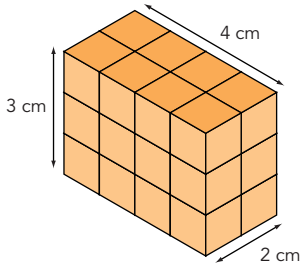
To find the volume, count the number of small cubes.

There are 8 small cubes on the top layer.
There are 8 small cubes on the middle layer.
There are 8 small cubes on the bottom layer.

Each small cube has a volume of 1 cm^3.



The volume of the shape is 24 cm^3.

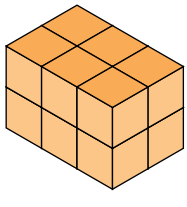


or length x width x height

4 cm x 2 cm x 3 cm = 24 cm^3

Question

Each small cube has a volume of 1 cm^3.
What is the volume of this shape?

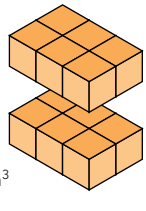


Answer

The top layer has 6 cubes.

The bottom layer has 6 cubes.

The volume of this shape is 12 cm^3.



or length x width x height

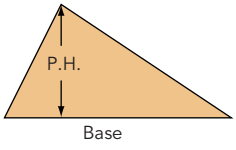
3 cm x 2 cm x 2 cm = 12 cm^3

Calculating length, area and volume – 1

You need to understand length, area, volume, perimeter and know the units each is measured in. You must know what is meant by cross-section, prism, parallelogram, trapezium, and how to use the formulae. (These formulae will be given on the examination paper.)

Remember: Perimeter is the distance around a shape.
Area is length x width (always measured in units^2, eg mm^2, cm^2, m^2)
Volume is length x width x height (always measured in units^3, eg mm^3, cm^3, m^3)

You are expected to know how to use these formulae:

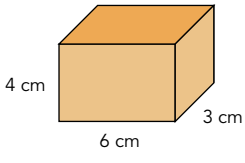


Area of a triangle
$= \frac{1}{2}$ x base x perpendicular height (P.H.)

Volume of a cuboid = length x width x height

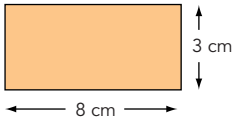
Example

Volume of a cuboid = 6 cm x 3 cm x 4 cm
= 72 cm^3

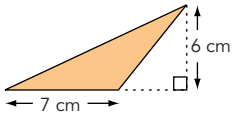


Questions

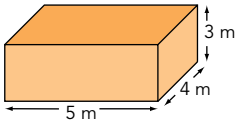
1 Find a the perimeter; and
b the area of this shape.



2 Find the area.



3 Find the volume.



Answers

1 a 8 cm + 3 cm + 8 cm + 3 cm = 22 cm
b 8 cm x 3 cm = 24 cm^2

2 $\frac{1}{2}$ x 7 cm x 6 cm = 21 cm^2

3 5 m x 4 m x 3 m = 60 m^3

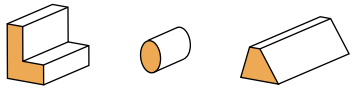
Calculating length, area and volume – 2

Prism

Any solid shape with uniform cross-section, ie same shape at each end.

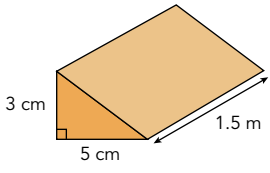
Cross-section

This is the shape that goes all through a prism, ie the shaded parts in these shapes.



Example

Find the volume of this prism:



Volume = cross-sectional area x length

First find the cross-sectional area

Area of a triangle = $\frac{1}{2}$ base x height = $\frac{1}{2}$ x 5 x 3 = 7.5 cm^2

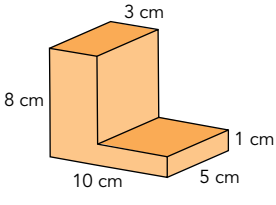
Note: The length is 1.5 m. This must be changed into centimetres, ie 150 cm.

Volume = 7.5 cm^2 x 150 cm = 1125 cm^3

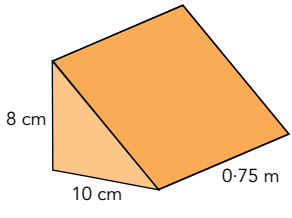
Questions

Find the volumes of these prisms:

1

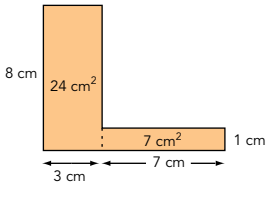


2



Answers

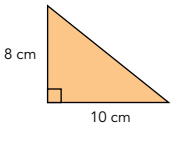
1 First find the area of cross-section:



Area of cross-section = 24 cm^2 + 7 cm^2 = 31 cm^2

Volume = area of cross-section x height
= 31 cm^2 x 5 cm
= 155 cm^3

2 Note the different units: metres and centimetres.
Change everything to centimetres [0·75 m = 75 cm]



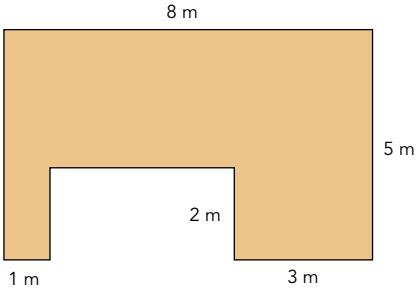
Area of triangle = $\frac{1}{2}$ x base x height
= $\frac{1}{2}$ x 10 x 8 = 40 cm^2

Volume = area of cross-section x height
= 40 cm^2 x 75 cm
= 3000 cm^3

Calculating length, area and volume – 3

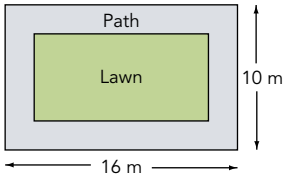
Questions

1 a Find the area.
b Find the perimeter.



2 This is a diagram of a garden with a lawn and a path around the edge. The path is 2 m wide.

Find the area of the path.

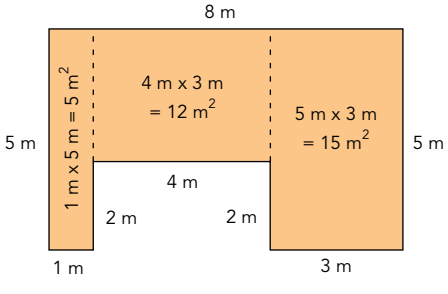


Answers

1 a Split the shape into three parts.

Area = 32 m^2

b 8 m + 5 m + 3 m + 2 m
+ 4 m + 2 m + 1 m + 5 m = 30 m



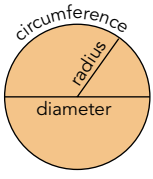
2 Find the area of the large rectangle = 10 x 16 = 160 m^2
Find the area of the small rectangle = 6 x 12 = 72 m^2
Take away = 88 m^2

Note: It is 6 x 12
A common error is 8 x 14.
Remember 2 m wide at both ends

You are advised to memorise the formulae for circles.

Formulae

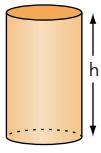
You must know how to use all of the formulae shown. Carefully note the two common errors at the bottom of the page.



Circumference of a circle = $2\pi r$
(this means 2 x π x radius)

Area of a circle = πr^2
(this means π x radius x radius)

Volume of a cylinder = $\pi r^2 h$
(this means π x radius x radius x height)



cylinder

A common error: Always ask yourself, does the question give the **radius** or the **diameter**? In any examination about 20% of the candidates will confuse radius and diameter. Be careful you are not one of them. To avoid this, before using any circle formulae ask "Do we have the radius?". If the answer is "Yes", continue. If not, find the radius. The **radius** is **half** of the **diameter**.

Questions

1 Find the circumference and area of a circle radius 6 cm
2 Find the circumference and area of a circle diameter 8 cm
3 Find the radius of a circle, circumference 20 cm
4 Find the volume of a cylinder diameter 80 cm, height 1·2 m

Answers

1 Do we have the radius? Yes. Continue.

Circumference = 2 x π x radius
= 2 x π x 6
= 37·7 cm

Area = π x radius x radius
= π x 6 x 6
= 113 cm^2

2 Do we have the radius? No. First we have to find the radius. The radius is 4 cm.

Circumference = 2 x π x radius
= 2 x π x 4
= 25·1 cm

Area = π x radius x radius
= π x 4 x 4
= 50·3 cm^2

3 Look back to *Equations: Rules* page 23.

Circumference = 2 x π x radius

$20 = 2 \times \pi \times r$

$\frac{20}{2} = \pi \times r$

$10 = \pi \times r$

$\frac{10}{\pi} = r$

3·18 cm = r

4 Do we have the radius? No. First halve the diameter to find the radius. The radius is 40 cm.

Volume = π x radius x radius x height
= π x 40 x 40 x 120
= 603186 cm^3
= 603000 cm^3

Common error: you cannot use mixed units, ie cm and m. Change 1·2 m into 120 cm

Tables and graphs

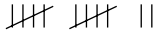
Again much of this is common sense. You need to be able to read information from tables and graphs in everyday life.

Frequency tables

You need to be able to use a tally system to produce a frequency. A tally is an effective way of counting.

When we complete a tally chart we count in fives.

 represents 5

 represents 12

Question

These are the numbers of sweets in 20 packets:

178	165	172	163	181
164	158	161	164	183
152	173	166	161	183
172	166	164	183	174

Complete this table to show the information.

Number of sweets in a packet	Tally	Frequency
150-159		
160-169		
170-179		
180-189		
	Total	

Answer

Use a pencil to cross out the original data as it is tallied.

Cross out 178 and record a tally in the tally column for 170-179. Cross out 165 and record a tally in the tally column for 160-169.

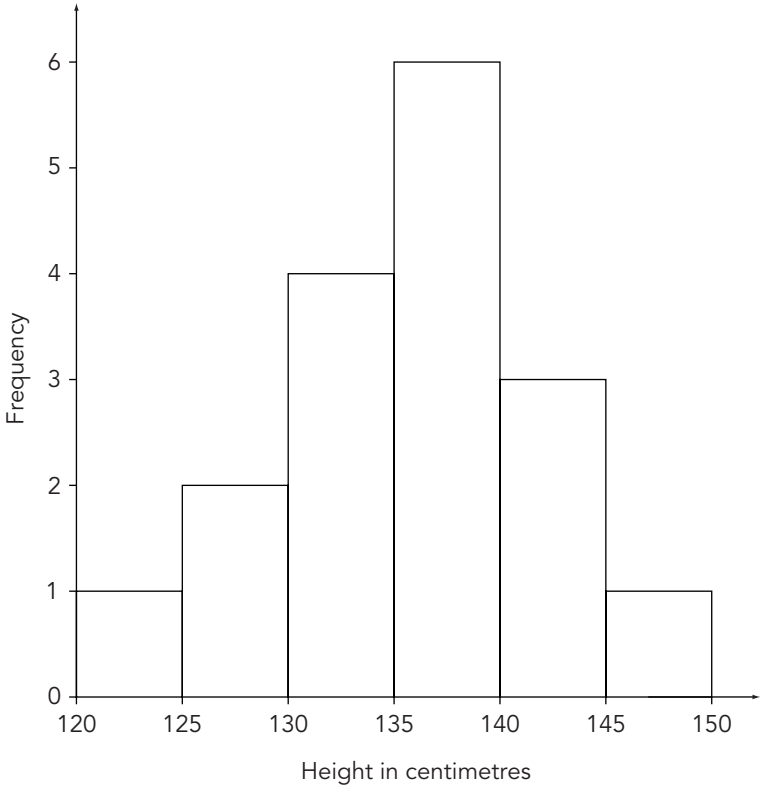
Number of sweets in a packet	Tally	Frequency
150-159	||	2
160-169		9
170-179		5
180-189	||||	4
	Total	20

When you have completed the table, check that the frequency total is correct. In this question there are 20 packets, so the total must be 20.

Frequency diagrams

You need to be able to understand and obtain information from a frequency diagram.

This is a frequency diagram. It shows the heights of pupils in a class.



Questions

1 How many pupils were between 130 and 135 centimetres?

2 How many pupils were between 120 and 125 centimetres?

3 How many pupils were between 125 and 140 centimetres?

4 How many pupils were under 130 centimetres?

5 How many pupils were in the class?

Answers

1 4

2 1

3 2 + 4 + 6 = 12

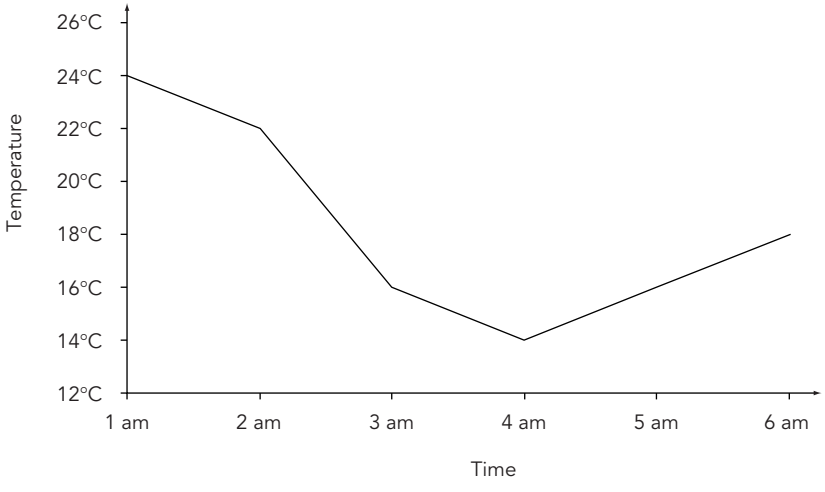
4 1 + 2 = 3

5 1 + 2 + 4 + 6 + 3 + 1 = 17

Line graphs

You will need to obtain information from line graphs.

This is a line graph. It shows the temperature in a room between 1 am and 6 am.



Questions

Use the line graph to answer the following questions:

1 What was the temperature at: a 2 am? _____ b 6 am? _____

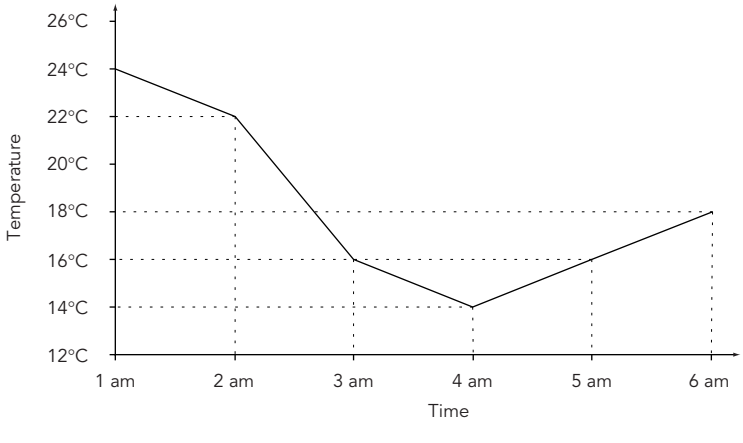
2 At what time was the temperature: a 16°C? _____ b 14°C? _____

Answers

Draw dotted lines on the graph as shown to find the answers.

1 a 22°C b 18°C

2 a 3 am and 5 am b 4 am



Using and drawing conclusions from graphs

Information can be obtained from graphs. You need to know how to extract the information you need.

This is a conversion graph for changing miles into kilometres.

Examples

1 The distance from Exeter to Dorchester is 50 miles. How far is this in kilometres?

Method: Find 50 miles on the graph. Draw a dotted line from the 50 mile mark to the conversion line. Draw a dotted line from the point it meets the conversion line to the kilometres scale. The distance in 80 kilometres.

2 Convert 300 kilometres into miles.

Method: The scale does not have 300 kilometres. Use 30 kilometres instead. 30 is about 19 miles. Therefore 300 kilometres is about 190 miles.

Questions

1 Convert 50 kilometres into miles.

2 Convert 30 miles into kilometres.

Answers

1 31 or 32 miles

2 48 kilometres

Your answers need not be exact.

Frequency tables and frequency diagrams

Continuous data is data which can have any value, eg distance between two places, height of a person. The height of a person can be measured to any degree of accuracy. A person could be 1.783642 m tall.

Discrete data is data which can only have certain values, eg the number of people in a room can only have whole number values. You cannot have 3.2 people in a room.

If you are asked to collect data you must choose an appropriate method. Usually a survey or an experiment. You must record your data and then present it in tables, diagrams and graphs.

Questions

The following are the times taken by 20 people to complete a jigsaw. The times are in minutes:

8.62, 28.4, 48.13, 30.1, 26.03, 47.42, 36.01, 25.23, 22.6, 29.97, 18.63, 30.00, 42.73, 38.62, 20.01, 19.99, 27.6, 16.32, 8.7, 12.58

a Record the information in a frequency table. Choose suitable equal class intervals.

b Show this information in a frequency diagram.

Answers

a A common error is:

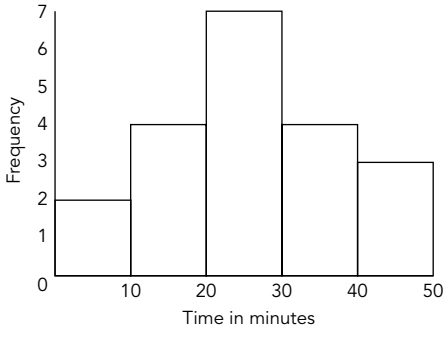
Minutes
0 - 10
10 - 20
20 - 30

Where would you record 20?
In the 10-20 or 20-30?

Minutes	Tally	Frequency
0 - under 10	II	2
10 - under 20	IIII	4
20 - under 30	~~IIII~~ II	7
30 - under 40	IIII	4
40 - under 50	III	3

Advice: Always add the frequency total. There are 20 people, therefore the frequency must add up to 20. → 20

b



Averages

There are three main types of average – mean, median and mode. You need to know each. Many students get them mixed up and that is just throwing away marks.

Median and mode

Median and mode are measures of average.

The **median** is the middle number when the numbers are placed in order.
The **mode** is the most common number.

Questions

1 Find the median and mode of these numbers:

2, 3, 5, 3, 2, 4, 2

2 Find the median of these numbers:

7, 3, 10, 2

3 The masses of boxers in a tournament are given in kilograms:

65, 63, 68, 64, 69, 68, 63, 64, 67, 69, 63, 61, 63, 67, 60

Find the median and the mode

Answers

1 First place the numbers in order of size

3 is the middle number, therefore the median is 3

2 2 2 3 3 4 5

There are more 2s than any other number, therefore the mode is 2

2 Place the numbers in order

2 3 7 10

The median is between 3 and 7

$\frac{3+7}{2} = \frac{10}{2} = 5$ The median is 5

3 Place the numbers in order: 60, 61, 63, 63, 63, 63, 64, 64, 65, 67, 67, 68, 68, 69, 69

The median is 64

The mode is 63

Mean and range

The **mean** is the most useful average because it uses all of the data. The mean is sometimes called the arithmetic mean.

The **range** is the difference between the largest and smallest numbers.

Example

a Find the mean of: 16, 18, 11, 19, 17 b Find the range

Method

a Add the numbers, then divide by how many numbers there are.

$$\frac{16 + 18 + 11 + 19 + 17}{5} = \frac{81}{5} = 16{\cdot}2$$

b The range is 19 – 11 = 8 [range = highest value – lowest value]

Questions

1 There are four children in a room. Their ages are: 16, 14, 13 and 15.

a What is the mean of their ages?

b What is the range?

2 This table show the number of letters delivered to houses in a street:

Letters	0	1	2	3	4	5
Number of houses	3	2	6	7	0	2

Calulate the mean number of letters delivered to each house.

3 The mean of four numbers is 7. The numbers are 5, 3, 8 and x. Find x.

Answers

1 a $\frac{16 + 14 + 13 + 15}{4} = \frac{58}{4} = 14{\cdot}5$

b 16 – 13 = 3

2 This is a very common exam question.

Two **common errors** are $\frac{0+1+2+3+4+5}{6} = 2{\cdot}5$ and $\frac{0+1+2+3+4+5}{3+2+6+7+0+2} = \frac{15}{20} = 0{\cdot}75$

$$\text{Mean} = \frac{\text{total number of letters}}{\text{total number of houses}} = \frac{(0 \times 3) + (1 \times 2) + (2 \times 6) + (3 \times 7) + (4 \times 0) + (5 \times 2)}{3 + 2 + 6 + 7 + 0 + 2} = \frac{45}{20} = 2{\cdot}25$$

3 The mean of four numbers is 7. Therefore, the total is 4 x 7 = 28.

$$5 + 3 + 8 + x = 28$$
$$16 + x = 28$$
$$x = 12$$

Comparing two sets of data

The mean, median or mode can be used as a measure of average.

If a question asks you to compare two lists of information, you must write about the differences between the lists.

If you have a choice, it is easiest to compare by using the mean. It is most difficult to compare by using the mode.

Question

These are the Maths test results (out of ten marks) for Jenny and Paul:

Test	1	2	3	4	5	6	7	8	9	10
Jenny's marks	8	6	8	5	4	6	7	6	8	4
Paul's marks	9	10	9	8	3	4	1	8	9	10

Use the range and mean to compare their marks. Who is better at Maths and why?

Answer

You should compare the ranges

Jenny's range of marks is 8 – 4 = 4

Paul's range of marks is 10 – 1 = 9

Jenny's marks have a smaller range. This suggests that she is more consistent than Paul. Jenny always gains a satisfactory mark. Paul scores some very good marks and some very poor marks.

Jenny's mean mark is $\frac{62}{10} = 6{\cdot}2$

Paul's mean mark is $\frac{71}{10} = 7{\cdot}1$

The mean marks suggest Paul is slightly better at Maths but the range suggests that he is very good in some areas and very poor in other areas.

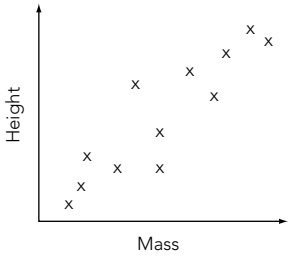
If a test question asks you who is better, you can state either person but you must give a reason based on the range and mean, median or mode.

Scatter diagrams

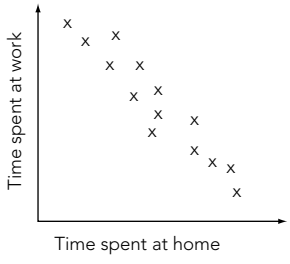
These are used to find connections between two sets of data.

Scatter diagrams

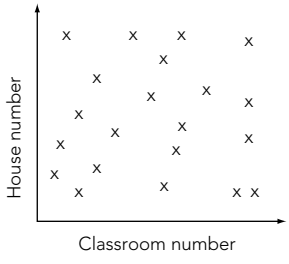
Scatter diagrams are used to find relationships (or correlation) between two sets of data.



This diagram shows a **positive correlation**



This diagram shows a **negative correlation**



This diagram shows **no correlation**

A positive correlation indicates that as one quantity increases so does the other quantity. The diagram shows that, in general, taller people are heavier.

A negative correlation indicates that as one quantity increases the other quantity decreases. The diagram shows that, in general, the more time a person spends at work, the less time they spend at home.

No correlation indicates that there is no relationship between the two quantities. The diagram shows that a house number has no connection with the classroom number.

> **Note:** Remember to use the word correlation in your answer.

Questions

1 Describe the relationship shown by this scatter diagram.

2 Explain the reason for this relationship.

Answers

1 Negative correlation.
As the temperature increases, the number of hot drinks sold decreases.
or As the temperature decreases, the number of hot drinks sold increases.

2 In hot weather people drink fewer hot drinks.
In cold weather people drink more hot drinks.

Pie charts

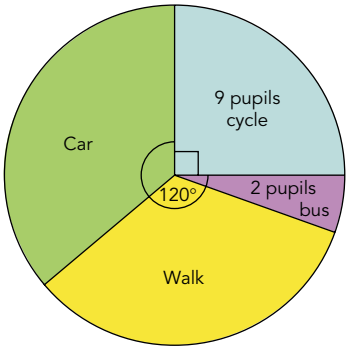
Look back at the angles section (see page 28). Pie charts allow us to present information. Information presented in a diagram is often easier to understand than information in a table.

Understanding pie charts

You will be expected to read information from pie charts and to draw pie charts.

Questions

This pie chart shows how the pupils in class 3A arrive at school:



1 How many pupils walk to school?

2 What is the angle for the bus sector?

3 How many pupils attend the school?

4 Complete the car sector

Answers

The first thing to do is find the angle for one pupil.
The pie chart shows 9 pupils cycle to school.
This sector is 90°.
9 pupils are represented by 90°
1 pupil is represented by 10°

1 The angle for the walk sector is 120°
We know that 1 pupil is represented by 10°
Therefore **12 pupils** are represented by 120°

2 2 pupils arrive by bus
We know that 1 pupil is represented by 10°
Therefore 2 pupils are represented by **20°**

3 There are 360° in a circle
We know that 1 pupil is represented by 10°
Therefore **36 pupils** are represented by 360°

4 The angles of a circle add up to 360°
cycle + bus + walk + car = 360°
90° + 20° + 120° + x = 360°
The angle for the car sector is **130°**
The pupils add up to 36
cycle + bus + walk + car = 36
9 + 2 + 12 + y = 36
13 pupils arrive by car

Drawing pie charts

Question

Thirty people were asked what sort of holiday they would choose. 5 said a mountain resort, 10 said a beach holiday, 7 said an activity holiday and 8 said a cruise. Show this information in a pie chart.

Answer

The first thing to do is find the angle for one person. There are 360° in a circle. The pie chart must represent 30 people.

360° ÷ 30 = 12° Therefore 12° represents 1 person.

Holiday choice	Frequency	Multiply by 12°	Angle at the centre of the pie chart
Mountain resort	5	x 12°	60°
Beach holiday	10	x 12°	120°
Activity holiday	7	x 12°	84°
Cruise	8	x 12°	96°

How to draw the pie chart

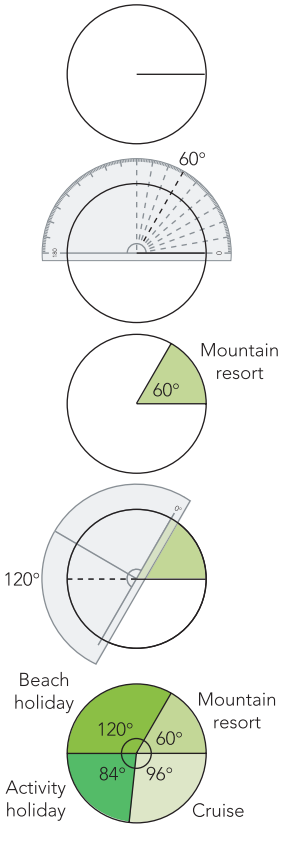
1 Draw a circle.
Draw a line from the centre to the edge.

2 Place the protractor on the circle.
Place the centre of the protractor on the centre of the circle.
Make sure 0° is on the line.
Measure the first angle, 60°.

3 Draw a line from the centre to the edge at 60°.
Label the sector "Mountain resort" and write 60°.

4 Move the protractor as shown.
Measure 120°.
Draw a line from the centre to the edge.

5 Repeat for 84°.
Check the remaining angle is 96°.
Label each sector.



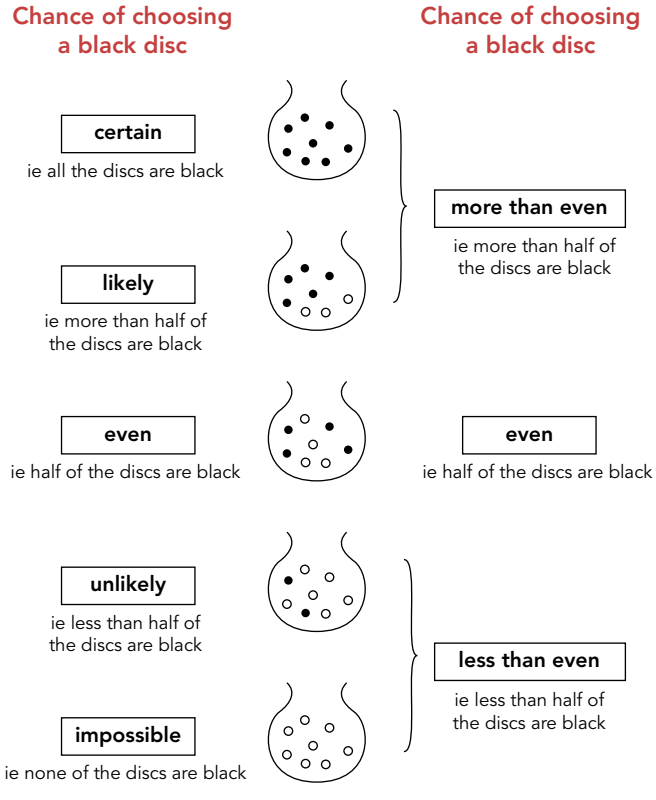
Do not forget: Label each sector and show the angle size.

Measure the angles carefully. If angles are not accurate, you will lose marks.

Probability is the chance of something happening.

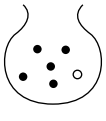
Probability

The words in boxes may be used in probability questions.



Questions

1 What is the probability of choosing a black disc from this bag?



Choose from certain, likely, even, unlikely, impossible.

2 How many discs of which colour should be added to give an even chance of choosing a black disc?

3 How many discs of which colour should be removed to make the chance of choosing a black disc certain?

Answers

1 Likely because more than half the discs are black.

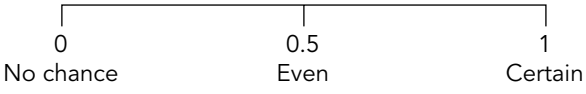
2 Add four white discs, then exactly half the discs will be black.

3 Remove one white disc. Then all the discs will be black.

The probability scale

The probability scale is used to show the chance of something happening. If something is impossible, eg picking a red disc from a bag of white discs, we say there is no chance and the probability is 0. If something will definitely happen, eg picking a white disc from a bag of white discs, we say it is certain and the probability is 1.

This is a probability scale:



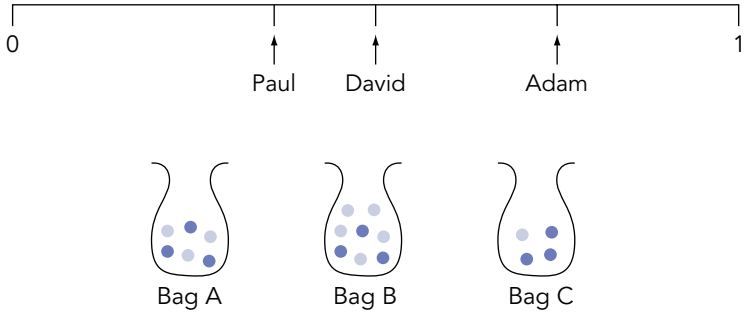
0 means no chance

1 means certain chance

0.5 means even chance

Questions

David, Paul and Adam are each blindfolded. Each chooses a ball from one of three bags. Their chances of choosing a black ball are shown on the probability scale.



Who chooses from:

1 Bag A
2 Bag B
3 Bag C?

Answers

1 3 out of 6 is an even chance. David chooses from Bag A.

2 3 out of 8 is less than an even chance. Paul chooses from Bag B.

3 3 out of 4 is more than an even chance. Adam chooses from Bag C.

Justifying probabilities

You can decide the probability of something happening by:

equally likely outcomes or experimental evidence.

We use equally likely outcomes for events with equal chances.

Eg: Throwing a die

The chance of throwing each number is equal. There are six numbers. The chance of throwing each number is $\frac{1}{6}$.

Sometimes events do not have equal chances. Then we must use experimental evidence.

Eg: The chances of a bus being late or on time are not equal.
We must observe and record the bus for a period of time.

Questions

1 Alan and Barry are in a race. Is it true that each boy has an equal chance of winning? If not, why not?

2 A coin is tossed. What is the chance of it landing on a head and why?

3 Toss a coin 20 times. Repeat the experiment five times. Do we always get the same number of heads and tails? Explain your answer.

Answers

1 It is very unlikely that each boy has an equal chance of winning. One of the boys is probably a better runner. The best way to decide the chances would be to observe the boys in, for example, ten races, ie experimental evidence. You would then know who was better.

2 There is an equal chance of a head or a tail, ie equally likely outcomes. The chance of a head is $\frac{1}{2}$.

3 We would expect an equal number of heads and tails each time. But this will not happen every time. Probability is a theoretical expectation. It is not a guarantee.

Probability (and, or)

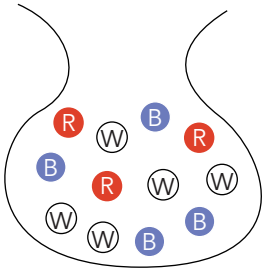
This page shows ways to calculate probablility.

A bag contains three red sweets, four blue sweets and five white sweets. A boy is blindfolded. He chooses a sweet. What is the probability he chooses:

a A red sweet?

b A blue sweet?

c A red sweet or a blue sweet?



Method

a $\frac{3}{12}$ ← There are three red sweets in the bag
← There are twelve sweets in the bag

b $\frac{4}{12}$ ← There are four blue sweets in the bag
← There are twelve sweets in the bag

c If a question states 'or' we must add

Probablilty of a red sweet or Probability of a blue sweet

$\frac{3}{12}$ + $\frac{4}{12}$

(use the fraction key $a\frac{b}{c}$ to add the fractions)

3 $a\frac{b}{c}$ 1 2 + 4 $a\frac{b}{c}$ 1 2 =

Answer $\frac{7}{12}$

Questions

1 What is the probability of choosing a white sweet?

2 What is the probability of choosing a red sweet or a white sweet?

Answers

1 $\frac{5}{12}$ ← There are five white sweets in the bag
← There are twelve sweets in the bag

2 Probability of a red sweet or Probability of a white sweet

$\frac{3}{12}$ + $\frac{5}{12}$

Calculator keys: 3 $a\frac{b}{c}$ 1 2 + 5 $a\frac{b}{c}$ 1 2 =

Answer $\frac{2}{3}$

Probability: Examination-type questions

Here are some examples of the questions you can expect to find on examination papers.

Questions

1 Show all of the possible outcomes when three coins are tossed.

2 a Complete this table to show all of the possible outcomes when throwing two dice.

	1	2	3	4	5	6
1						
2						
3						
4						
5						
6						

b How many different ways can two dice land?

c What is the probability of a double?

3 The probability of a new light bulb not working is 0.03. What is the probability of a new light bulb working?

Answers

1 HHH HHT HTH HTT TTT TTH THT THH

2 a

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

b 36 ways

c There are 6 doubles

There are 36 different ways

Probability = $\frac{6}{36} = \frac{1}{6}$

$\frac{6}{36}$ ie $\frac{\text{number of ways required}}{\text{total number of ways}}$

3 A light bulb can either work or not work so the total probability is 1.

Probability of working	+	probability of not working	=	1
?	+	0·03	=	1
Probability of working			=	1 – 0·03
Probability of working			=	0·97

Questionnaires

This may be useful for your coursework. It is quite easy. Just use your common sense.

Designing questionnaires

Questionnaires are used to obtain information. You may need to design a questionnaire as part of your coursework.

1 Design your questions to obtain information you can present and analyse in a variety of ways. A variety of different ways to present your data is given on pages 54-64.

2 Make your questions easy to understand.

3 Do not ask embarrassing questions, eg "How many boyfriends do you have?"

4 Provide a choice of answer, eg "Do you do a lot of homework?", will produce answers such as "yes", "sometimes", "only in Maths". These responses are difficult to present and analyse. A better question would be:

"How much time did you spend doing homework last night? Tick the box nearest to the amount of time."

0 hours ☐ 1 hours ☐ 2 hours ☐ 3 hours ☐

Types of question

Your questionnaire should contain one or two questions of each of the following types:

1 Questions with yes/no responses, eg "Do you own a bicycle?" Yes ☐ No ☐

Try to avoid questions to which everyone will answer yes or everyone will answer no. Your results can be shown as a percentage, in a bar graph, pictogram, pie chart, etc.

2 Questions with numerical answers, eg "How many televisions do you have in your house?"

Your results can be presented in graphs, tables, etc. You can calculate the mean, median and mode of the data.

3 Questions you can compare, eg "What was your percentage mark in the English exam?" and "What was your percentage mark in the Maths exam?"

These questions will allow you to draw a scatter diagram to test a hypothesis such as "Pupils who obtain high marks in English also obtain high marks in Maths."

How many people to ask

Twenty is a good number. Each person represents 5% of the total and each person can be represented by 18° on a pie chart.

Forty is a good number. Each person represents 2.5% of the total and each person can be represented by 9° on a pie chart.

How many questions to ask

A maximum of ten.

Diagnostic tests

These tests will help you check how good you are at questions on each topic.
If you have difficulty, revise the topic again.

1 Place value

1 Write seventeen thousand and twenty in figures.

2 Place the following numbers in ascending order:
1103, 1036, 997, 96 121, 3748

2 Multiplication facts

1 What is the missing number?

a $6 \times \square = 42$

b $\square \div 3 = 7$

c $54 \div \square = 6$

2 Work out the following:

a 72×100

b $5800 \div 10$

3 Negative numbers

1 Place these numbers in order, smallest first: -1, 0, 3, -5, 2

2 $-8 + 3 =$

3 $-5 - 2 =$

4 Solving problems without a calculator (do not use a calculator)

1 Work out the answers without using a calculator. You must show all of your working.

a
$$\begin{array}{r} 48 \\ \times\ 3 \\ \hline \end{array}$$

b $96 \div 4$

2 A hall had eight rows of chairs with seventeen chairs in each row. What was the total number of chairs?

5 Calculation checks (show your working)

1 Estimate:

483 + 214 + 689 + 307 + 593

2 Write 78·362 correct to the nearest whole number.

3 Write 614·72 correct to the nearest whole number.

6 Mental arithmetic shortcuts – 1

1 $600 \times 80 =$

2 $5600 \div 70 =$

7 Mental arithmetic shortcuts – 2

1 $42{\cdot}3 \times 100 =$

2 $3{\cdot}82 \div 1000 =$

8 Decimals

1 $3{\cdot}6 + 42 + 0{\cdot}38 =$

2 $8{\cdot}3 - 2{\cdot}71 =$

3 $5{\cdot}62 \times 0{\cdot}03 =$

4 $4{\cdot}2 \div 0{\cdot}06 =$

9 Long multiplication and division

1 523×74

2 $665 \div 17$

10 Checking

1 Write out a subtraction sum to check this sum:

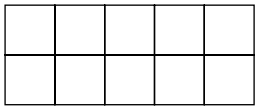
$$\begin{array}{r} 732 \\ +\ 189 \\ \hline 921 \end{array}$$

2 Estimate the cost of 407 tapes at £3.95 each.

11 Fractions and percentages

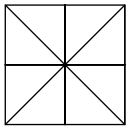
1 a Shade $\frac{2}{5}$ of the shape below.

b What fraction is unshaded?



2 a Shade 25% of the shape below.

b What percentage is unshaded?



12 Fractions

1 Find $\frac{5}{16}$ of 4

2 72 people out of 88 watched *Neighbours*. Write this as a fraction in its lowest terms.

3 $5\frac{3}{4} \div 1\frac{2}{3}$

13 Changing between decimals and percentages

1 Write 27·3% as a decimal.

2 Write 0·038 as a percentage.

14 Changing between decimals, percentages and fractions

1 Convert 0·024 to a fraction.

2 Write $\frac{3}{8}$ as a decimal.

3 Write $\frac{4}{5}$ as a percentage.

15 Ratio – 1

1 Simplify the ratio 36:27

2 The scale of a map is 1:50 000. The distance from Hilton to Longden is 12 cm on the map. What is the actual distance? Give your answer in kilometres.

16 Ratio – 2

1 A sum of money was divided between Alan and Barry in the ratio 3:5. Alan received £150.

a How much did Barry receive?

b What was the total amount of money?

17 Percentages

1 14 people out of 80 wore a hat. Write this as a percentage.

2 A man bought a painting for £500. Two years later he sold it for £650. Calculate the percentage gain.

3 A woman bought a vase for £60. She sold it for £48. Calculate the percentage loss.

18 Calculating percentage parts

1 Find 27% of 32

2 Mr Soames earns £240 per week. He receives a 4·2% pay rise. How much is his pay rise?

19 Patterns you must recognise

1 What are the special names given to these numbers?

a 1, 4, 9, 16, 25, 36...

b 1, 8, 27, 64, 125, 216...

c 1, 3, 6, 10, 15, 21...

2 List the factors of 30.

3 Complete the next three prime numbers:

2, 3, 5, 7...

20 Using formulae

This is a formula:

Cost of hiring a cement mixer = delivery charge + (charge per day x number of days)

1 Calculate the cost of hiring a cement mixer if the delivery charge is £30 and the mixer is hired for six days at £10 per day.

2 The cost of hiring a cement mixer was £70. The delivery charge was £30 and the charge per day was £10. How many days was the cement mixer hired?

21 Writing in algebra

1 A cake costs y pence. What is the cost of seven cakes?

2 S sweets are divided equally between four children. How many sweets does each child receive?

22 Using algebra

1 Find the value of the following formulae, when x = 6, y = 4, z = 2

a 3x b 2y – z

c 3xy

2 T = AB + 4C. Calculate T when A = 4, B = 3 and C = 2

23 Rules

Solve these equations:

1 y + 7 = 10 2 y – 3 = -8

3 6y = 27 4 $\frac{y}{4} = 3$

24 Writing equations

A man buys x books at £7 each. The total cost is £63.

1 Form an equation to show this

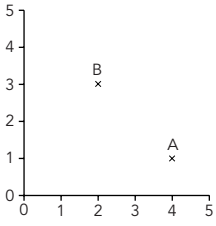
2 Solve the equation

25 Trial and improvement

1 $x^3 = 104$.

Find the value of x correct to one decimal place using trial and improvement. You must show all of your working.

26 Co-ordinates



1 Give the co-ordinates of the following points:

a A b B

2 Mark these points on the graph:

a Y is the point (1, 2)

b Z is the point (3. 1)

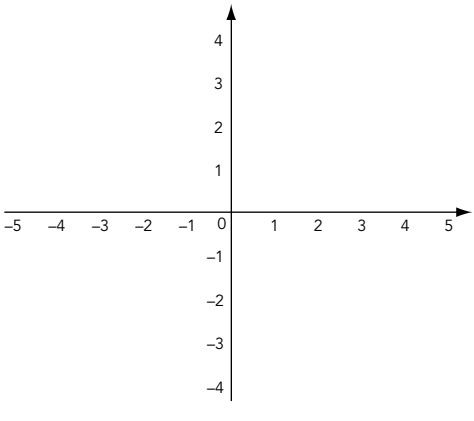
27 Drawing lines

Draw and label the following lines

1 y = 0 2 x = 0

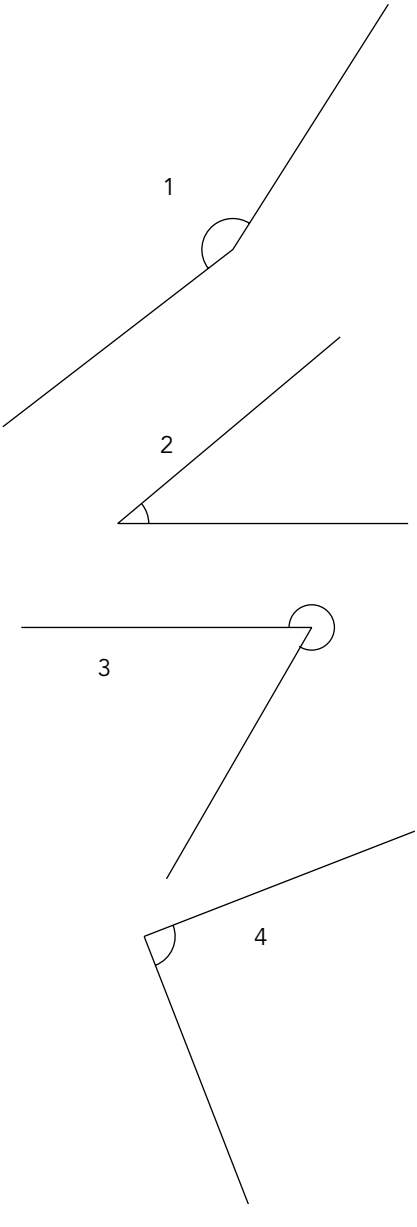
3 y = 4 4 x = -3

5 y = x 6 y = -x



28 Using a protractor

Use a protractor to measure these angles.



29 Angles: Acute, obtuse, reflex

Look at the angles for question 28 again. Name them. Choose from:

acute angle obtuse angle

right angle reflex angle.

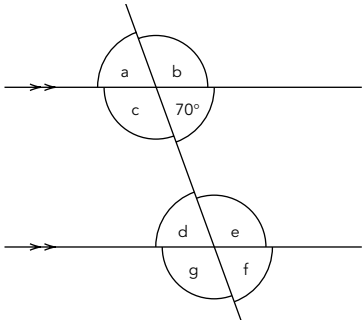
30 Intersecting and parallel lines

Look at the diagram below.

1 Find the size of the missing angles.

2 Angle c and angle e are

3 Angle a and angle d are



31 Regular polygons

1 Find the size of each exterior angle of a regular pentagon.

2 Find the size of each interior angle of a regular pentagon.

3 A regular polygon has an exterior angle of 18°. How many sides does it have?

32 Bearings

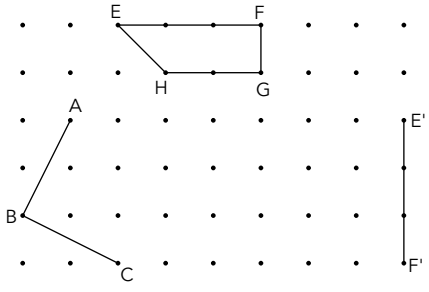
In the diagram below, A, B and C are three ships.

1 What is the bearing of A from B?

2 What is the bearing of B from A?

3 What is the bearing of B from C?

33 Common 2-D shapes



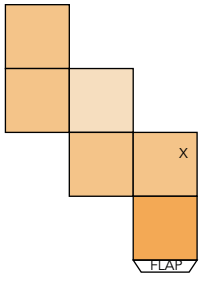
1 ABCD is a square. Complete the diagram.

2 The shape EFGH is turned and moved. E moves to E′. F moves to F′. Complete the diagram and mark the points G′ and H′.

34 2-D representations of 3-D shapes

1 Name the 3-D shape formed by this net:

2 This net will form a cube:

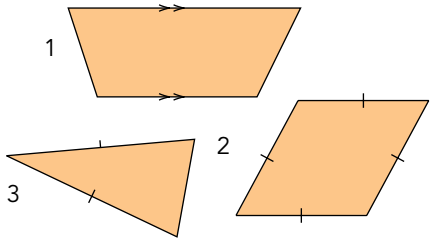


a Place an x in each corner which touches x when the cube is formed.

b Mark F on the edge to show where the flap will fit when the cube is formed.

35, 36 Properties of quadrilaterals and triangles

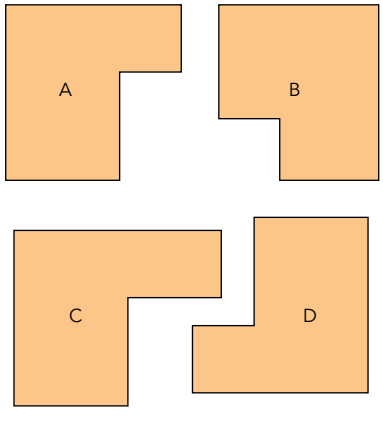
Name these shapes:



4 Complete this diagram to make a kite.

37 Congruent shapes

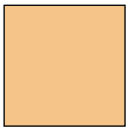
Below are four shapes. Which two are congruent?



38 Rotational symmetry

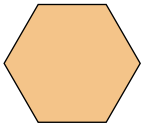
Look at these shapes. If a shape does not have rotational symmetry write "no rotational symmetry". If it does have rotational symmetry, write the order and mark the centre of rotation with an x.

1

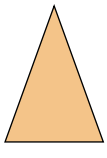


2

3



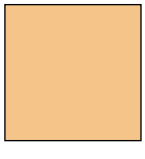
4



39,40 Symmetry of 2-D shapes

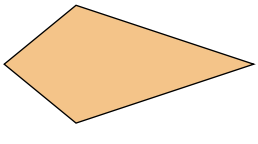
Draw the axes of symmetry on these shapes.

1

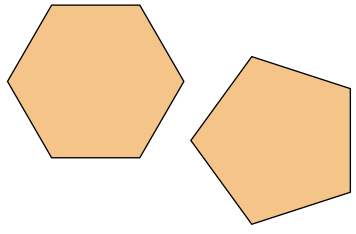


2

3



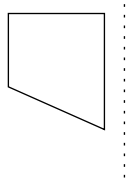
Draw the axes of symmetry of these shapes.



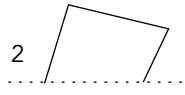
41 Reflection

Reflect the following shapes in the mirror line:

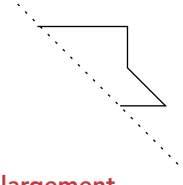
1



2

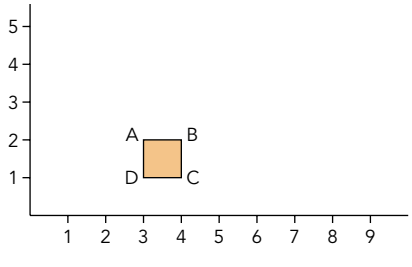


3



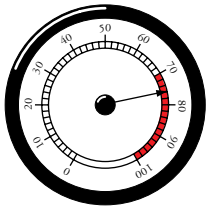
42,43 Enlargement

Enlarge the square ABCD by a scale factor of 3. Centre of enlargement is the point (2,1).



44 Metric units of measure

1 This is a car speedometer. It shows the speed in kilometres per hour. What speed is shown?



2 Which of these units would you use to measure the distance from London to Manchester? Choose from millimetres, kilometres, metres, kilograms, tonnes, centimetres.

45 Rough metric equivalents of Imperial units

1 My garden is 20 yards long. How many metres is this approximately?

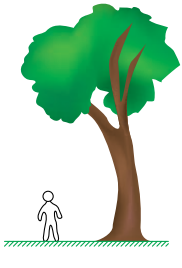
2 I bought ten pounds of potatoes. How many kilograms is this approximately?

46 Converting one metric unit to another

1 Convert 0.36 kilometres into metres.

2 Convert 850 millilitres into litres.

47 Making sensible estimates



This man is standing by a tree. Estimate the height of the tree.

48 Time

1 A car travels for 6 hours 47 minutes. It arrives at its destination at 17:24. What time did it begin?

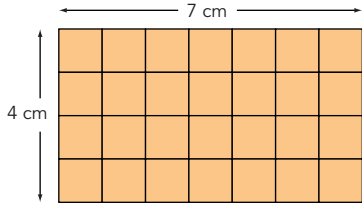
2 A train leaves Poole at 08:27 and arrives in London at 10:14. How long did the journey take?

49 Perimeter, area and volume

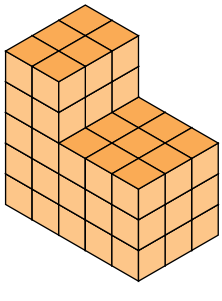
1 Each small square has an area of 1 cm^2. What is:

a The perimeter of this shape?

b The area of this shape?

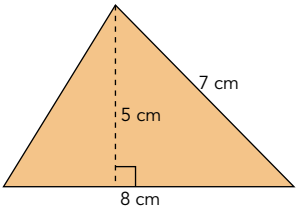


2 Each small cube has a volume of 1 cm^3. What is the volume of this shape?



50 Calculating length, area and volume – 1

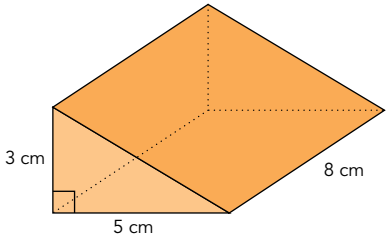
1 Calculate the area:



2 The volume of a cuboid is 144 cm^3. The length is 8 cm, the width is 6 cm. Calculate the height.

51 Calculating length, area and volume – 2

Find the volume of this shape.

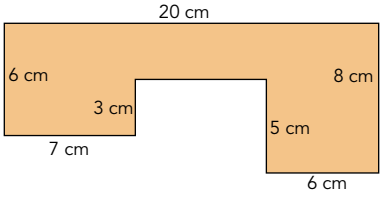


52 Calculating length, area and volume – 3

Find:

1 the area and

2 the perimeter of this shape.



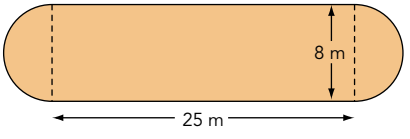
53 Formulae

1 Find the:
a circumference and
b area of a circle, diameter 30 cm.

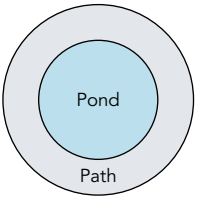
2 The circumference of a circle is 70 m. Calculate the radius.

3 Calculate the volume of a cylinder radius 4 cm, height 7 cm.

4 Find the:
a area and
b circumference of this shape.



5 The diameter of this circular pond is 10 m. A path, 3 m wide, goes all the way around. Find the area of the path.



54 Frequency tables

These are the numbers of drawing pins in each of 20 packets:

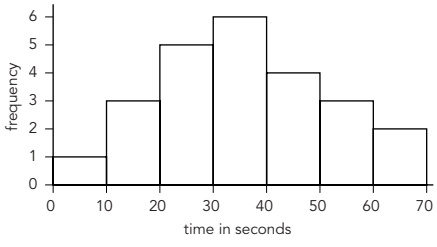
523 517 503 527 537 535 513
514 523 537 519 530 515 523
525 508 532 529 524 516

Complete this table to show the information

Number of drawing pins in a packet	Tally	Frequency
500 - 509		
510 - 519		
520 - 529		
530 - 539		
	Total	

55 Frequency diagrams

This is a frequency diagram. It shows the time (in seconds) taken by different people to thread a needle.



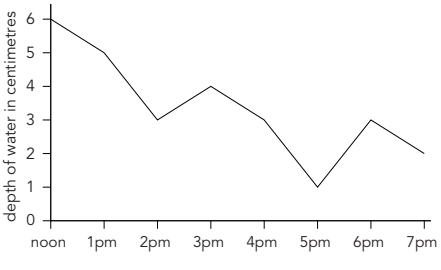
1 How many people took between 20 and 30 seconds?

2 How many people took less than 20 seconds

3 How many people attempted to thread a needle?

56 Line graphs

This is a line graph. It shows the depth of water in a stream. Readings were taken every hour.



Use the line graph to answer the following questions:

1 At what time was the depth

a 4 cm b 6 cm

2 What was the depth at

a 6pm b 1pm

57 Using and drawing conclusions from graphs

This is a conversion graph to change gallons into litres:

1 Convert 9 litres into gallons.

2 Convert 3 gallons into litres.

3 Convert 40 gallons into litres.

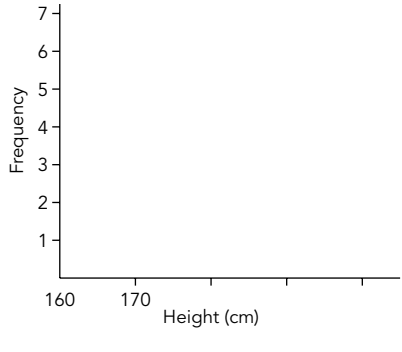
58 Frequency tables and frequency diagrams

This data shows the height of 20 adults. Height is in centimetres.

163	178	179	168	180
179	187	165	183	178
174	193	193	184	193
184	168	187	189	193

Complete the frequency table and frequency diagram below.

Height	Tally	Frequency
160-under 170	IIII	4
170-under 180		



59, 60 Mean, median, mode and range

Find the mean, median, mode and range of:

1 5, 8, 12, 5, 6

2 3, 10, 4, 10, 3, 5, 14, 12, 10, 6

61 Comparing two sets of data

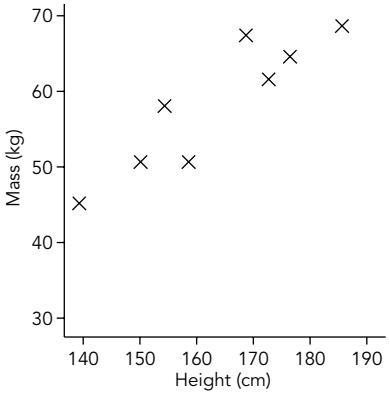
This table shows the weight (in kg) of two flocks of sheep:

Flock A	Flock B
30 45 70	52 58 64
68 52 38	80 73 74
57 78 82	76 56 62
38	74

Use the range and mean or median to compare the two flocks.

62 Scatter diagrams

This scatter diagram shows the height and mass of eight girls aged 15:

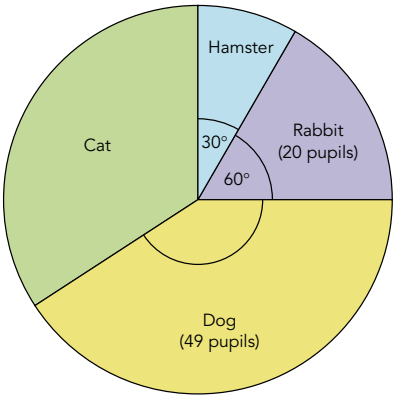


1 Describe the relationship shown by this graph.

2 What is the mass of the girl who is 150 cm in height?

63 Understanding pie charts

This pie chart shows the favourite pets of Year 11 pupils.



1 How many pupils are in Year 11?

2 How many pupils chose "hamster"?

3 What is the angle for "dog"?

4 How many pupils chose "cat"?

5 What is the angle for "cat"?

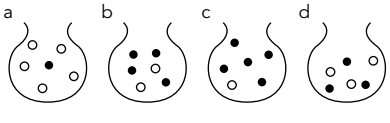
64 Drawing pie charts

Show this information in a pie chart:

Favourite sport	Number of pupils
Swimming	7
Fishing	12
Tennis	12
Football	9

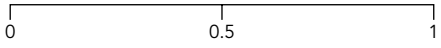
65 Probability

1 Decide for each of the following bags if the chance of choosing a black disc is less than even, even, or more than even.



66 The probability scale

Show the probability of the following events on the probability scale:



1 Shaking a six on a die. Mark with an A.

2 Shaking an even number. Mark with a B.

3 Shaking a number greater than 7. Mark with a C.

67 Justifying probabilities

How would you work out the probability of each of the following? Choose equally likely outcomes or experimental evidence.

1 How a coin will land.

2 How a drawing-pin will land.

3 Whether Alan or Barry is the faster runner.

68 Probability (and, or)

A bag contains four red discs, four blue discs and two yellow discs. A girl is blindfolded and selects a disc. What is the probability of selecting:

1 A red disc?

2 A yellow disc?

3 A red or a yellow disc?

69 Probability: Examination-type questions

The probability of a new battery failing is 0.003.

a What is the probability of a new battery working?

b 40 000 batteries were produced. Estimate how many failed.

70 Designing questionnaires

1 State one advantage and one disadvantage of asking this question:

Which is your favourite subject?

2 State one advantage and one disadvantage of asking this question:

Place a tick by your favourite subject from this list	
Maths	
English	
French	
History	
Science	

Answers to diagnostic tests

1

1 17 020

2 997, 1036, 1103, 3748, 96 121

2

1 a 7 b 21

c 9

2 a 7200 b 580

3

1 -5, -1, 0, 2, 3 2 -5

3 -7

4

1 a $\begin{array}{r} 48 \\ \times\ 3 \\ \hline 144 \\ {\scriptstyle 2} \end{array}$ b $\begin{array}{r} 24 \\ 4\overline{)9^{1}6} \end{array}$

2 136

5

1 500 + 200 + 700 + 300 + 600 = 2300

2 78 3 615

6

1 48 000 2 80

7

1 4230 2 0·00382

8

1 45·98 2 5·59

3 0·1686 4 70

9

1 38 702 2 39 remainder 2

10

1 $\begin{array}{r} 921 \\ -189 \\ \hline 732 \end{array}$ or $\begin{array}{r} 921 \\ -732 \\ \hline 189 \end{array}$ 2 £1600 (400 x 4)

11

1 a Shade any four squares b $\frac{6}{10}$ or $\frac{3}{5}$

2 b Shade any two sections b 75%

12

1 $1\frac{1}{4}$ 2 $\frac{9}{11}$

3 $3\frac{9}{20}$

13

1 0·273 2 3·8%

14

1 $\frac{3}{125}$ 2 0·375

3 80%

15

1 4:3 2 6 km

16

1 a £250 b £400

17

1 17·5% 2 30%

3 20%

18

1 8·64 2 £10·08

19

1 a Square numbers

b Cube numbers

c Triangle numbers

2 1, 2, 3, 5, 6, 10, 15, 30

3 11, 13, 17

20

1 £90 2 4 days

21

1 7y or 7 x y

2 $\frac{5}{4}$

22

1 a 18

b 6

c 72

2 20

23

1 y = 3 2 y = -5

3 y = 4·5 4 y = 12

24

1 7x = 63

2 9

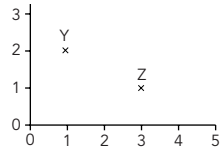
25

1 4·7

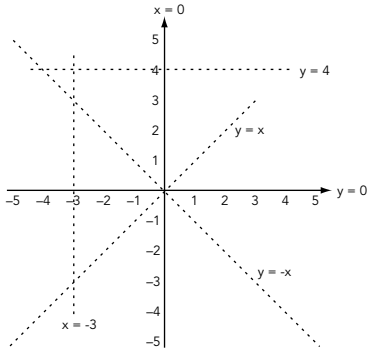
26

1 a (4, 1) b (2, 3)

2



27



28

1 160° 2 40°

3 300° 4 90°

29

1 Obtuse 2 Acute

3 Reflex 4 Right angle

30

1 a = 70° 2 Alternate

b = 110° 3 Corresponding

c = 110°

d = 70°

e = 110°

f = 70°

g = 110°

31

1 72° 2 108°

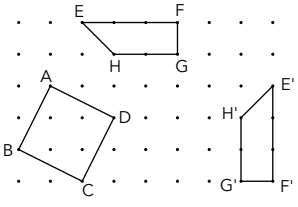
3 20

32

1 286° 2 106°

3 056°

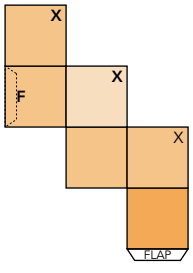
33



34

1 Triangular prism

2

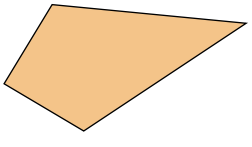


35, 36

1 Trapezium 2 Rhombus

3 Isosceles triangle

4

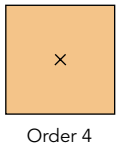


37

A and D

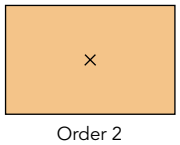
38

1



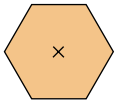
Order 4

2



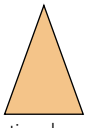
Order 2

3



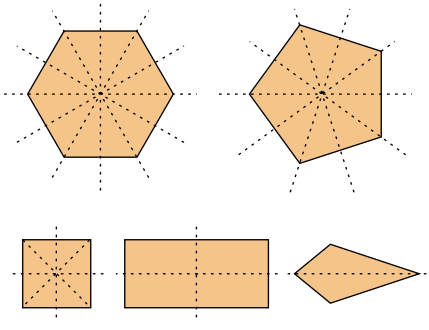
Order 6

4

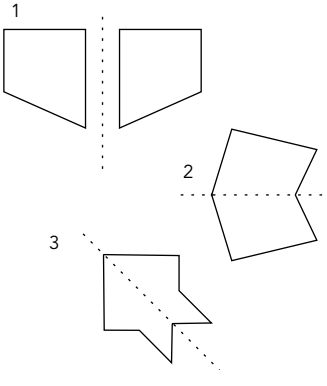


No rotational symmetry

39,40



41



42, 43

A′ (5,4), B′ (8,4), C′ (8,1), D′ (5,1)

44

1 76 km/h 2 kilometres

45

1 17-20 m 2 4-5 kg

46

1 360 m 2 0·85 litres

47

Any answer between 8 m and 12 m

48

1 10:37

2 1 hour 47 minutes

49

1 a 22 cm b 28 cm^2

2 57 cm^3

50

1 20 cm^2

2 3 cm

51

1 60 cm^3

52

1 111 cm^2

2 62 cm

53

1 a 94·2 cm b 707 cm^2

2 11·1 m 3 352 cm^3

4 a 250 m^2 b 75·1 m

5 Area of large circle = 201·06 m^2

Area of small circle = 78·54 m^2

Subtract 201·06 – 78·54

Answer = 122·5 m^2 (approx)

54

Number of drawing pins in a packet	Tally	Frequency
500 - 509	II	2
510 - 519	𝍸 I	6
520 - 529	𝍸 II	7
530 - 539	𝍸	5
	Total	20

55

1 5 2 4

3 24

56

1 a 1:30 pm and 3 pm b noon

2 a 3 cm b 5 cm

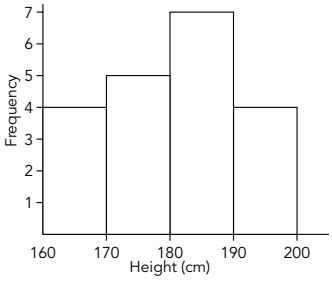
57

1 2 gallons 2 13·5 litres

3 180 litres

58

Height	Tally	Frequency
160-under 170	IIII	4
170-under 180	~~IIII~~	5
180-under 190	~~IIII~~ II	7
190-under 200	IIII	4



59, 60

1 Mean = 7·2, median = 6, mode = 5, range = 7

2 mean = 7·7, median = 8, mode = 10, range = 11

61

Flock A	Flock B
range 52	range 28
mean 55·8	mean 66·9
median 54·5	median 68·5

The range of flock B is lower than flock A. This shows flock B is more consistent with less variation in the weight of sheep in the flock.

The mean of flock B is higher than flock A. This suggests flock B is heavier.

The median of flock B is higher than flock A. This suggests flock B is heavier.

62

1 Positive corellation, or the taller the heavier

2 50 kg

63

1 120 2 10

3 147° 4 41

5 123°

64

360 ÷ 40 = 9° per pupil

swimming = 63°, fishing = 108°, tennis = 108°, football = 81°

65

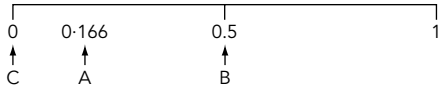
1 a less than even

b more than even

c more than even

d even

66



67

1 Equally likely outcomes

2 Experiment

3 Experiment

68

1 a $\frac{4}{10} = \frac{2}{5}$

b $\frac{2}{10} = \frac{1}{5}$

c $\frac{6}{10} = \frac{3}{5}$

69

1 a 0·997

b 120

70

1 It is an open-ended question. Pupils can state their favourite subject (advantage). There may be a large number of different responses. These may be difficult to analyse and record (disadvantage).

2 It is a closed question. The subject chosen is only the favourite from the list, so it may not be the pupil's favourite (disadvantage). There are only five possible responses. This will make it easy to analyse and record the results (advantage).

Index

Ii

Kk

Ll

Mm

Nn

Oo

Pp

Qq

Rr

Ss

Tt

Uu

Vv

Ww

Xx

Yy

Zz

123